Booming sands and granular physics

Clark Ritz
Carleton College,

Booming sands and granular physics

Clark Ritz
Senior Integrative Exercise
Carleton College

ABSTRACT

Booming and squeaking sands produce incredible acoustic emissions when they avalanche (in the case of booming sands) or are struck or compressed (in the case of squeaking sands). Understanding this phenomenon could lead to advances in other areas of granular physics, such as the problem of treating granular flows with fluid mechanical models. This paper gives a brief overview of several topics in granular physics, reviews experimental work done to determine the properties of booming sands, and outlines two theories of the physical mechanism that produces acoustic emissions, the first mechanical, the second fluid mechanical.

I. INTRODUCTION

Certain desert and back beach dunes throughout the world produce incredible, other worldly acoustic emissions when they avalanche. As of 1997, at least 31 booming dunes had been identified from California to the Kalahari,[1] which produce sounds likened to drums, low-flying propeller driven aircraft, thunder, and other notably loud occurrences. The dominant frequencies of the sounds produced range from 50-200 Hz for this booming sand.[2] Sand known as squeaking, sonorous musical, or frog sand produces a squeaking sound with a dominant frequency of 500-1500 Hz when sheared at roughly 45° from normal, as when stepped on or struck with a rod.[2] Though they have been recorded for at least 1500 years[3] and studied for over a century[4], the underlying physics are still unclear.

At this point, the question remains, "Why does one study booming sand at all?" As Peter Haff, a noted researcher in the field of granular physics, states, "To understand booming is to understand much of the yet poorly investigated field of granular mechanics."[5] Physicists, geologists, and engineers study the properties and mechanics of booming sand in the hope of shedding light on the broader area of granular physics. This increasingly popular field of study is far more important than the casual observer may realize. "The processing of granular media and aggregates," as Duran, another important granular researcher, points out, "consumes roughly 10% of all the energy produced on this planet," and is second only to the processing of water in importance to humans.[6] Given the great investment of energy and recourses, even a modest increase in efficiency in processing granular matter would have dramatic economic impacts due to the enormous scale of any possible applications.

Though important, the physics of granular materials is complicated, nonlinear, and still not well understood. I will begin with an introduction to the difficulties of granular physics and the classical mechanics used to develop it. From there I will cover a few more specific topics of interest: packing; pressure and force distribution; the stick-slip effect and avalanching; fluid-dynamical descriptions of granulars; and size segregation. I will then review experimental work done to date on booming sands and discuss the properties that differentiate them from normal sand. From there, I will give an overview of two theories, one strictly mechanical and one fluid mechanical in nature, of the actual mechanism that is suspected of producing the acoustic emissions in both booming and squeaking sands.

II. GRANULAR PHYSICS

It seems as though the physics of such ordinary materials composed of small, individually well understood solid particles would be just as understandable when aggregates of them are examined; however, granular physics is far from simple and well understood. This section outlines the problems faced by researchers in this field, explains some of the physical models used to describe granular materials, and discusses some unusual properties of granulars. A good introductory text (also the only introductory text) on granular physics is the book by Duran, from which many of the following developments and figures are adapted or taken.[6]

A. Difficulties in dealing with granular material

Researchers cannot simply extend theories developed to explain the behavior of solids, liquids, and gases because granular matter is distinctly different from any other state of matter.[7] Unlike solids, granular aggregates do not transmit forces homogeneously and their density varies with the packing configurations of individual grains. The length and time scales over which fluid-dynamical equations are valid are much too small to be applied to granular flow. Finally, necessarily inelastic collisions between grains preclude the application of the kinetic theory of gases. The highly energetic fourth state of matter, plasma, is clearly a poor model for a placid pile of sand. The inability to apply previously developed theories is quite marked, and can be easily demonstrated.

Following is an attempt to use a simple approximation, whose failure will show that a granular material cannot be approximated as a gas by treating each individual grain as if it were a molecule in a fluid.[6] We will make the assumption that the behavior of a sand pile can be understood by treating its constituent sand grains as the molecules of a gas. We will start with classical gravitational potential energy of the form $E_p = mgh$, where E_p is the energy, m is the mass of the sand grain, h is the height of a sand grain above level ground, and g is the gravitational acceleration at the earth's surface. We will equate this E_p to the thermal energy of a body, typically given, by $E = kT$, where k is Boltzmann's constant and T is the temperature of the body. Equating the potential and the thermal energy and solving for T using typical values of m for sand, $m = 300\mu\text{g}$ and assuming $h = 300\mu\text{m}$, a typical diameter for a grain of sand, we find $T \approx 10^{15}\text{K}$. This value of T is far larger than temperatures that a sand pile would experience under normal conditions on earth, which suggests to us that it is unfeasible to simply approximate a macroscopic granular aggregate as a gas by equating the individual grains with microscopic particles.

B. Classical mechanical concerns

Classical mechanics serves as the basis of granular physics, although an infinitely powerful computer and intimate knowledge of each and every constituent particle would be needed to exactly solve the equations of motion for all grains involved. Dry friction, in the form $F_f = \mu N$, between two objects sliding on one another plays an important role, where F_f is the magnitude of the frictional force (exerted in the direction opposite the sliding motion), N is the normal force between the object experiencing F_f and the surface exerting the force on it, and μ is the coefficient of friction, which varies with the materials concerned and is different for static and dynamic situations. For a high friction situation, say a block of wood on sandpaper, μ is large in contrast to a low friction situation, like a hockey puck on an a sheet of ice, for which μ would be small. Below in FIG. 1 of a block sliding across a level surface it can be seen that the frictional force will act in the opposite direction to the applied force F.

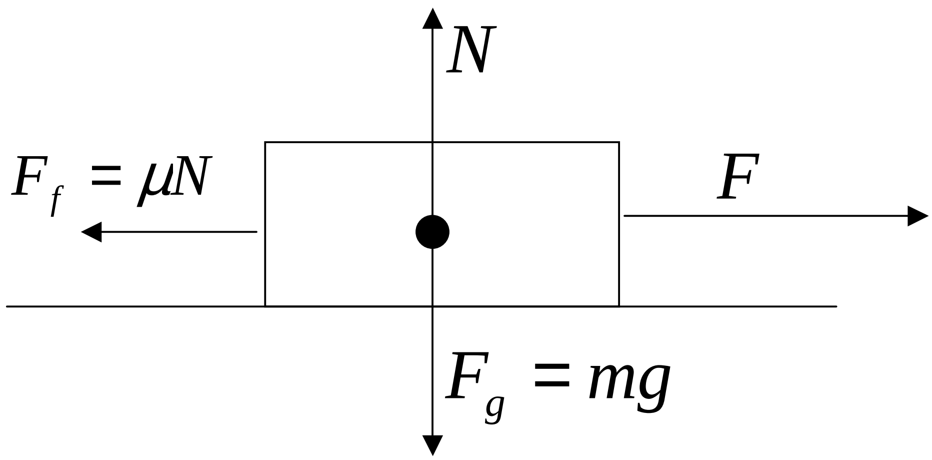
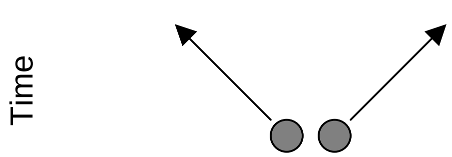


FIG. 1. Here, the force F accelerates a block of mass m to the right across a level surface, but the frictional force, F_f, in the opposite direction, counteracts this acceleration. F_g is the weight of the block.

Basic kinematics and conservation laws are also useful when describing granular behavior. Also important are the dynamics of individual grain-grain collisions, which are complicated due to the facts that the collisions are between finite objects, that collisions can be off center, that they can involve rotations of the particles due to frictional interactions, and that they are inelastic. The coefficient of restitution, $0 \le \varepsilon \le 1$, is used to measure the inelasticity of the collisions. If $\varepsilon = 0$ the collisions are totally inelastic and all kinetic energy is converted to sound waves, heat, or other forms of energy (see the one dimensional collision in FIG. 2). If $\varepsilon = 1$the collision is perfectly elastic and no kinetic energy is lost (see the one dimensional collision in FIG. 2). We will be seeking models that can describe an entire collection of grains, but we will refer back to this level of grain-grain and grain-surface interaction throughout the following discussions.



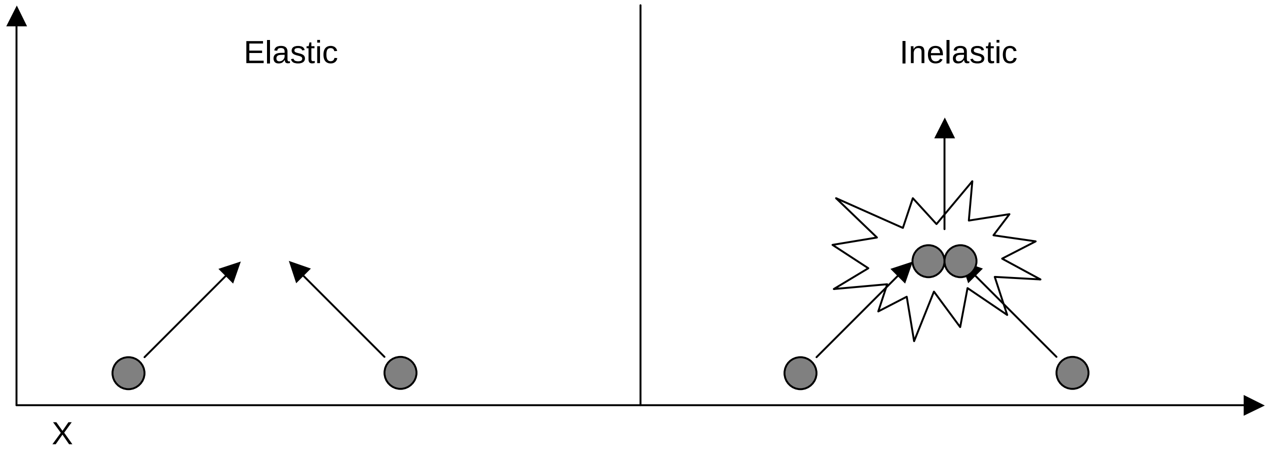


FIG. 2. On the left, the spheres taking part in the totally elastic collision lose no kinetic energy, whereas the totally inelastic collision on the right releases the kinetic energy of the spheres as sound or other forms of energy (represented by starburst pattern in the figure) and remain stationary after the collision.

C. Packing of a granular material

The behavior of a granular aggregate is highly dependent upon its packing and compressibility, and it is useful to define and examine parameters that describe them. To describe packing, first consider a unit cell of four identical circles of radius *R*, which are continuously in contact, and a parallelogram drawn connecting the centers of the circles, shown in FIG. 3. We are going to observe the total area of the figure *S*, the area of all four spheres plus the area of the void between them, as defined in this equation:

$$S = 3\pi R^2 + \frac{h_h h_v}{2}, \tag{1}$$

and track the change in this area, ΔS, as the formation of the circles is deformed. The diagonals of the parallelogram, h_h and h_v, are related by the following equation do to the restriction that the spheres must always remain in contact:

$$h_h^2 + h_v^2 = 16R^2. \tag{2}$$

This allows us to write ΔS as

$$\Delta S \approx \frac{h_h}{2}\sqrt{16R^2 - h_h^2} = 2h_h R\sqrt{1-\frac{h_h^2}{(4R)^2}}. \quad (3)$$

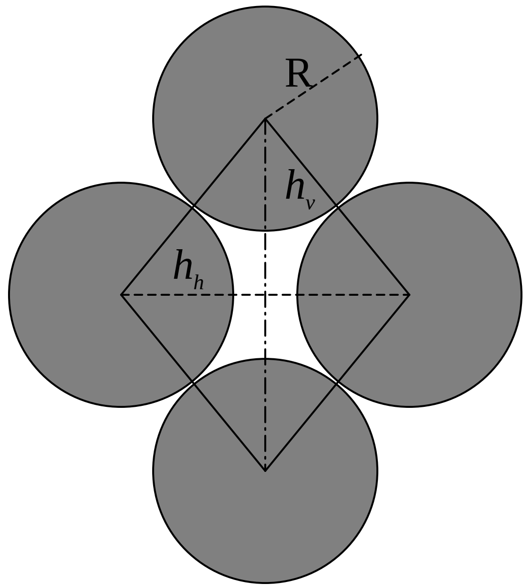


FIG. 3. Formation of four identical circles and inscribed parallelogram connecting their centers, which is used to study the packing configuration of granular materials.[6]

The figure below, FIG. 4., is a normalized plot of ΔS as a function of h_h, that is, the change in the collective area of the circles and the void between them as the length of the horizontal diagonal is changed by some squeezing or stretching force. (Remember that the circles are constrained such that they must remain in contact with one another without deforming.) At the left endpoint the left and right circles are in contact, and the top and bottom circles are maximally separated. Correspondingly, at the right endpoint the top and bottom circles are in contact and the left and right circles are maximally separated. The most notable feature of the plot is the maximum. This means that, if we begin in the configuration signified by the left endpoint and compress the cell from the left and right, we will see the collective area of the figure increase until we reach the maximum. In this regime, a counterintuitive principle is at work; a compressive stress actually increases area. This principle is known as Reynolds's dilatancy principle,[6] and it states that well compacted granular materials will expand when compressed. After reaching the maximum seen in the plot, further compression along the horizontal axis of the figure will again decrease the area, which is the response of an ordinary solid to compression. These regimes are shown as regions on the graph in FIG. 4.

This effect is observed quite readily when one is walking on wet sand. The foot shearing the packed sand causes it to expand, or rather, to shift into a different packing configuration that takes up more total volume and that leaves more interstitial space into which water can drain. This is the result seen above, and it depends on the various lattices into which grains can pack themselves. In two dimensions, a triangular lattice is the optimal way to pack circles, and any shearing of the grains in this configuration (shown on the left in FIG. 5) will cause them to move into another, less compacted configuration (shown right in FIG. 5).

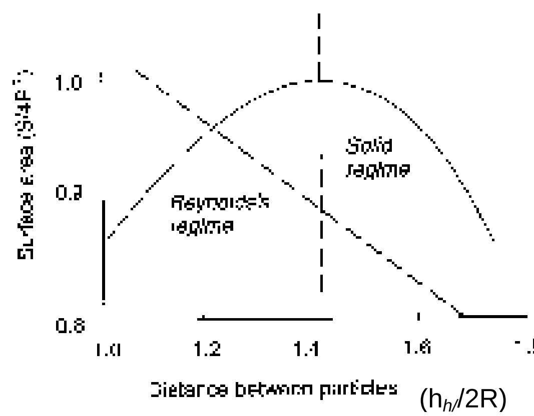


FIG. 4. Normalized plot of the surface area of a configuration of four identical circles versus the normalized length of the horizontal diagonal. Close-packed configurations of circles (see FIG. 3) "dilate," or grow larger if they are compressed when they are already in an optimized packing.[6]

Tighter packing

Loser packing

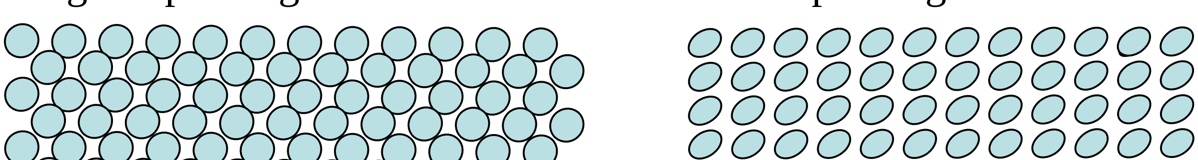


FIG. 5. Demonstrates tightest packing versus looser packing of circles. If the stack of circles on the left were sheared into the configuration on the right, its total area would expand.

D. Pressure and force distribution

In addition to Reynolds's property of dilatancy, granular materials exhibit another unusual property; the nonlinear distribution of forces. Granular materials are not continuous and the actual grain-grain contacts are distributed randomly, and, therefore, forces applied to an aggregate of grains are transmitted nonlinearly through these random grain-grain contacts. Below, FIG. 6 depicts the random distribution of forces through grain-grain contacts by optically distinguishing the grains that are under greater stress. Two crossed polarizers are placed on both sides (one behind and one in front) of the beads, and would block all light from passing through the material but for a phenomenon called stress birefringence.[8] Because of this phenomenon the way in which the beads themselves polarize different wavelengths of light changes as stresses are applied to them, making the beads that are subject to stress visible. The random, dispersive nature of force distribution has interesting consequences for the behavior of a granular material when subject to applied forces.

Water and other normal fluids stored in a vertical container under gravity will experience a pressure gradient, increasing with depth, due to the weight of the fluid at the top of the container pressing down onto the fluid below, in contrast to granular material.

In normal fluids, any differential portion of the fluid will experience a pressure corresponding linearly to the height and, therefore, the mass of fluid directly above it. Pressure in a granular material is not linearly depth dependent. The nonlinear, random force chains shown in the FIG. 6 are responsible for this effect. The downward force of gravity on a given grain is not transmitted from the given grain to a grain directly below it; there may not even be a grain directly below it. Instead, the force exerted on the grain by gravity is transmitted to the other grains that are randomly in contact with the grain being considered and eventually to the container walls. This property has been exploited for thousands of years in the hourglass.[9] Pressure at the connection between the two halves of an hourglass, and therefore the rate of sand flow, is relatively independent of the height of sand in the top half (to a good approximation) and serves as a linear measure of time. Many other important instantiation of this phenomenon occur in industry and agriculture, as in a grain silo for example.

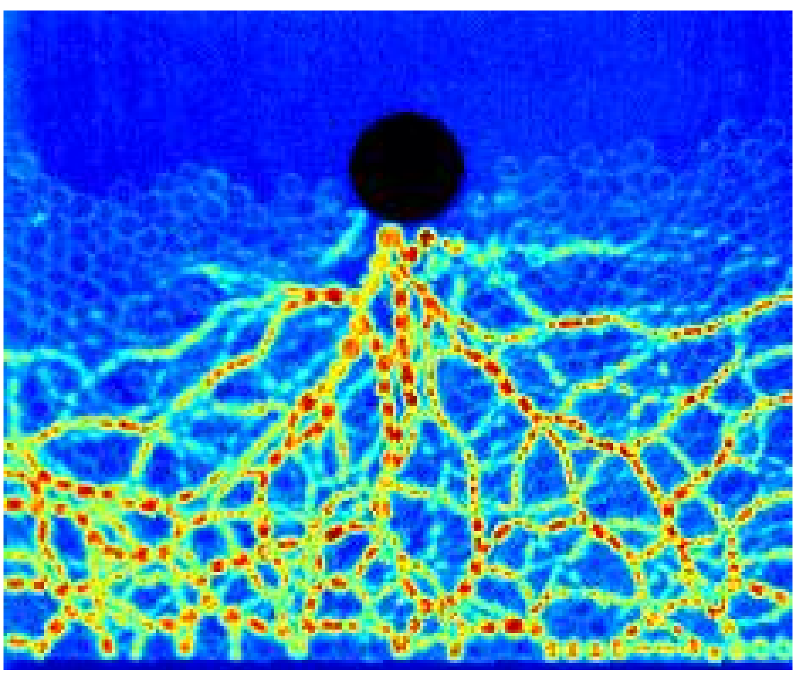


FIG. 6. This image depicts the distribution of stresses of the impact of the larger, dark ball throughout the granular medium. Grains shown in red are subject to greater stress and make up the force chains that distribute stresses. Stress birefringence and polarizers are used to create this image.[10]

In the late 1800's Janssen developed a model of the distribution of stress through a granular material in a vertical silo-like container. Janssen's model, also elaborated upon by Lord Rayleigh, is based upon solving a differential equation relating the vertical and horizontal pressure at a given point in the granular medium and also upon several important assumptions and simplifications.[6] First we assume that the medium is continuous, which, from a mathematical point of view, is a valid simplification if a large number of grains are considered and size of the system being investigated is much larger than an individual grain. We also ignore local rotations of particles, ignore interactions with the interstitial air, and concentrate on the long-term behavior of the system. Second, we assume, in the case of this model, that an applied vertical force, p_v, creates a horizontal force, p_h, such that $p_h = Kp_v$, where K is a constant of proportionality.

Now we can describe a physical system and arrive at a differential equation, which, when solved, will illustrate the nonlinear height dependence of pressure in granular materials. As shown in the FIG. 7 below, we will consider a slice dh of a granular material in a roughly cylindrical container with a base of area A and a perimeter P. The vertical coordinate, h, is set to zero at the top and increases toward the bottom of the container. The slice experiences an upward force of $A\,dp_v$ (pressure increases with depth) and its own weight, $\rho gAdh$, where ρ is the constant density of the slice and g is the acceleration of gravity. The third and final force that the slice feels is an upward frictional force from the walls due to infinitesimal downward movements. Janssen and Rayleigh ignored the randomness of grain-grain contact points (as well as the resulting randomness of frictional forces) and assumed that frictional force on the slice followed the form $F = \mu_s N$ where μ_s is the coefficient of friction for the grains with the surface of the container and *N* is the normal force that is exerted by the surface of the container, the horizontal force in this case. Above, we assumed a relationship p_v and p_h in the container, and we will use it to replace p_h in the frictional term. Because we are interested in finding the total upward frictional force on the slice, we will need to take into account the area of the slice, *Pdh*, in contact with the walls of the container. Thus, the frictional force on the slice is $K\mu_s p_v Pdh$.

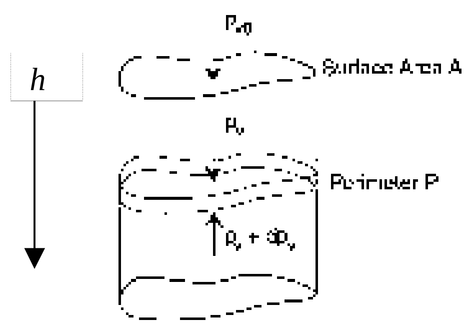


FIG. 7. Somewhat generalized silo system and parameters used in Janssen's model. We consider a slice *dh* of a granular material at equilibrium in a silo-like container.[6]

In equilibrium, with all three forces on the slice equal (the force of gravity towards the ground, the frictional force toward the top of the container, and the additional normal force towards the top of the container from the mass below the slice), we find the following relation:

$$Adp_v + K\mu_s p_v Pdh = \rho g A dh. \qquad (4)$$

Dividing through by *A* and *dh*, we come to the differential equation

$$\frac{dp_v}{dh} = +\left(K\mu_s \frac{P}{A}\right) p_v = \rho g \qquad (5)$$

We then rewrite equation (5) into the form

$$\frac{d}{dh}\left[\exp\left(K\mu_s\frac{P}{A}\right)p_v\right]=\rho g\exp\left(K\mu_s\frac{P}{A}h\right), \tag{6}$$

and integrate to find

$$p_v\exp\left(K\mu_s\frac{P}{A}\right)=\rho g\exp\left(K\mu_s\frac{P}{A}h\right)+C. \tag{7}$$

The constant C can be found if we create an initial force, $p_{v0}=Mg/A$, by placing a mass M on top of the container over its base area. Using these initial conditions, we solve for C and finally arrive at an equation for $p_v(h)$:

$$p_v=\rho g\frac{A}{PK\mu_s}\left[1-\exp\left(-K\mu_s\frac{P}{A}\right)\right]+p_{v0}\exp\left(-K\mu_s\frac{P}{A}h\right). \tag{8}$$

If we use equation (8) to study the behavior of p_v as h increases from zero, we find that, for small values of h, p_v initially increases linearly with h, just as water would. Quickly though, as h becomes large, we see p_v saturate and asymptotically approach a maximum value, as shown in FIG. 8 below. Beyond this point, p_v is independent of h. We see the linear regime in which the granular material behaves as a normal liquid would, followed by the asymptotic regime in which granular materials take on this characteristic, nonlinear behavior of constant pressure regardless of column height. The model developed in this subsection is general for silos, and can be adapted for differing geometries.

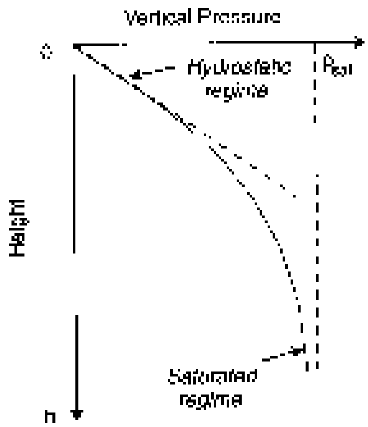


FIG. 8. This plot demonstrates the vertical pressure in a column of granular material as a function of height of the column. The hydrostatic regime, in which pressure varies linearly with height, and the saturated regime, in which the pressure is independent of the height of the column of the granular material can clearly be seen.[6]

E. Stick-slip effect and avalanching

The stick-slip effect is a very commonly observed wherever elasticity and friction both play a role and it also offers an interesting model for avalanching observed in rotating drum experiments. In the following section, I will first qualitatively explain the stick-slip effect, rotating drum experiments, and critical angles. Then I will develop a model used to describe both avalanching and the stick-slip effect and point out their similarities.

Friction and elasticity in concert are responsible for the stick-slip phenomenon, which can be described with a fairly simple example shown in FIG. 9 below. Think of a block at the origin, which is attached to an anchored, un-stretched spring along the x direction and placed on a conveyor belt that, via frictional forces, will pull the block against the spring in the +x-direction. The static frictional force will be constant, and the force of the spring will increase as the block moves away from the origin. At some point, the spring force will overcome the static frictional force and pull the block along the conveyor belt, which will pull the block along against the kinetic frictional force until the force of the spring on the block is less than the kinetic frictional force, which will then begin to accelerate the block back in the direction of the motion of the belt. We end up with the block oscillating in the x-direction. This type of motion is also responsible for the vibrations produced by the bowing of a violin string, squeaking hinges, and screeching fingernails on a chalkboard.

Seemingly far removed from a mass and a spring, granular avalanches have strange behaviors of their own, most notably the occurrence of two critical angels and the participation of only parts of the aggregate in avalanching. These two important angles are θ_r, the angle of repose, and θ_m, the angle of motion. When the face of a granular pile is inclined at θ_r it is in a stable state and neither vibrations nor other perturbations will cause an avalanche. Angles between θ_r and θ_m are metastable, meaning that a granular pile inclined to an angle in this range will remain stationary if left unperturbed but will avalanche if perturbed. A sand pile with a slope that exceeds θ_m cannot be constructed. Granular materials piled up past θ_m or inclined past θ_m will necessarily avalanche. It is further significant that any avalanching pile will avalanche only until it reaches θ_r and then will stop. The increase in the angel of the sand from θ_r to θ_m without avalanching is analogous to the stretching of the spring without dragging the mass in the mass-spring system. The values of these angles vary for different types of grains and are typically found experimentally. Most measurements of sand find $\theta_r \approx 35°$ and $\theta_m \approx 37°$. It is also important to note that only the top layers of a pile are in motion during an avalanche, as shown in FIG. 10.

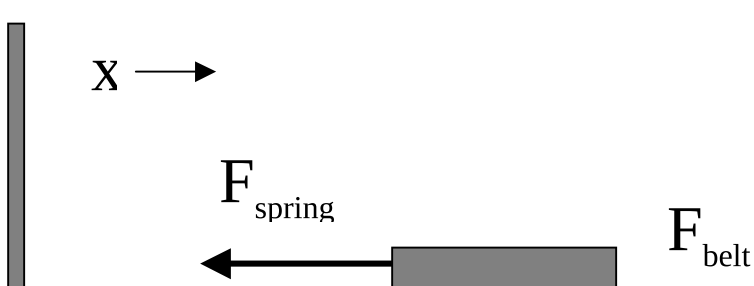


FIG. 9. Diagram of stick-slip illustration. The belt carries the block to the right via frictional force, while the spring pulls to the left. Adapted from Duran.[6]

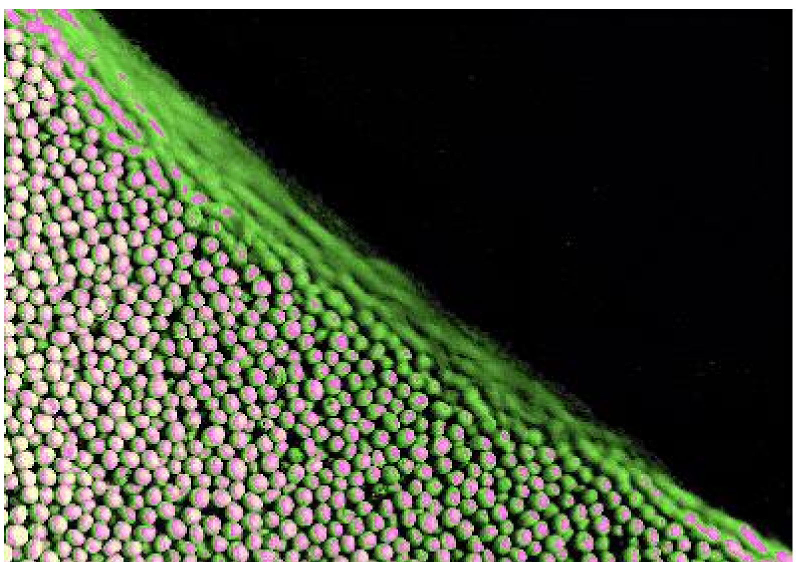


FIG. 10. Mustard seeds induced to avalanche. Only the top layers of the heap are avalanching. Cover April 1996 *Physics Today*.

A rotating drum experiment is frequently used to observe these types of behavior. A drum with a window on one end, or some other means of observing the contents, is partially filled with a granular material and rotated along its axis (See FIG. 11a). The angle that the surface of the aggregate makes with the surface can be observed, as well as the flux of material. This type of apparatus can be used for many different types of experiments, but we are only interested in stick-slip avalanches at this point. Imagine a drum filled with material whose surface is initially parallel to the horizon. As the drum rotates, the surface of the material will eventually reach θ_m, avalanche back down to θ_r, and, due to the drum's continued rotation, eventually reach θ_m again and avalanche a second time. Continued, steady rotation of the drum leads to a continual and periodic switching between static, or stick, and dynamic, or slip, states in the avalanching material. The correspondence of these states to the stick and slip of a mass on a spring is more than qualitative.

The relevant physics and mathematics describing the stick-slip motion of a mass dragged across a rough surface by a spring can describe similar states of motion in avalanching grains in a rotating drum. Both systems are depicted in FIG. 11. Perhaps surprisingly, all of the quantities used to describe one system have analogues in the other, as shown in Table 1. In fact, the same differential equation describes both systems.

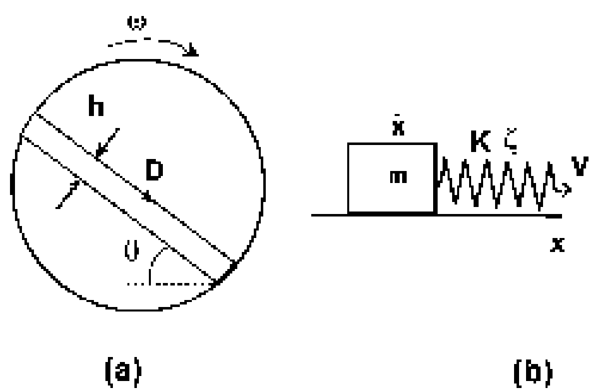


FIG. 11. Depictions of stick-slip models for (a) intermittent avalanching of granulars in a continuously rotating drum, and (b) for the jerky, start-stop motion of a mass dragged across a surface with friction by a spring.[6]

Rotating Drum Variables	Stick Slip Variables
θ	ζ
D	$\dot{x}$
ω	V

Table 1. These variables describe the relevant observables and parameters of the rotating drum model of avalanches and the mechanics of a mass pulled by a spring across a rough surface. The same differential equation can describe both systems when the above substitutions are made.[6]

The differential equations describing the systems is based on the mechanics of the mass spring system, but it also describes the behavior of avalanching in a rotating drum if the appropriate corresponding observables and parameters are substituted. Describing the mass/spring system are ζ, the deformation of the spring, $\dot{x}$, the velocity of the mass, and *K'*, the spring constant. Correspondingly, θ, the angle of the avalanching material makes with the horizon, *D*, the flux of material, and ω, the angular velocity of the drum, describe the avalanching system. The differential equation for the mass/spring system is derived from basic mechanics of a harmonic oscillator in motion around its equilibrium, where the equilibrium is defined by $K'\zeta = mg\mu_d$. The mass concerned is *m*, *g* is the acceleration of gravity, and μ_d is the dynamic coefficient of friction. We find:

$$m\frac{d^2\zeta}{dt^2} + K'\zeta = mg\mu_d\left(V - \frac{d\zeta}{dt}\right). \tag{9}$$

Of course, there is also a different, static coefficient of friction, μ_s, which results in a stick slip effects. This gives us the conditions for slipping: $\left(d^2\zeta/dx^2\right) = 0$, $\left(dx/dt\right) = 0$, and $K'\zeta < mg\mu_s$.

A phase space diagram of the relevant variables, depicted in FIG.12 clearly shows the static and dynamic properties of the systems concerned with an ellipse and a flat line respectively. The elliptical portion of the diagram corresponds to the acceleration and deceleration, or slip, of the block being pulled by the spring or to the flowing of sand in the rotating drum. The flat line at the bottom of the figure corresponds to the stick portion of the cycle in which the spring stretches while the block remains stationary or to the portion or the drums rotation in which the sand does not flow as its face angle increases from θ_r to θ_m. Measurements taken of actual rotating drum experiments trace similar patterns in phase space, lending credence to this type of model.

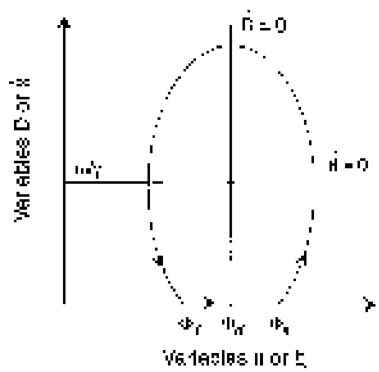


FIG. 12. Phase space diagram of variables present in mass/spring model and rotating drum model. The elliptical portion of the graph corresponds to the intensifying and slacking of movement/sand flow, and the flat line at the bottom of the path corresponds to the stick portion of the cycle during which no avalanching or movement of the mass occur.[6]

F. Fluid-dynamical description

The qualitative similarity of flow of granular materials to the flow of a normal, Newtonian fluid suggests that the physical descriptions of fluid mechanics as we know them should be applicable to granular flow. In fact, modifications have been made to fluid-dynamical equations in the hopes of describing granular flows in which the separation distance between the individual grains, *s*, is much less than *D*, the diameter of the grains, but non-zero. In a much cited article, Haff proposed heuristic equations for granular flow based on continuous matter fields, conservation principles, and macroscopic variables, which revert to grain-grain interactions only when necessary.[11] Haff's approach and a few other issues relating to a fluid mechanical description of granular matter are described here only qualitatively due to the complexity of the mathematics.

The equations stem from conservation of energy and momentum and take the form of a continuity equation, a momentum equation, and an energy equation. Conservation of mass serves as the basis for the continuity equation. The momentum equation is a modification of the notoriously difficult Navier-Stokes equation, and the energy equation also essentially takes the hydrodynamic form. In this granular description, however, the hydrodynamic coefficients like viscosity, thermal diffusivity, and energy absorption due to collisions are functions of the local state of the medium rather than constants. The equations resulting from this approach are nonlinear and coupled, but can be solved analytically for some simplified static and steady state conditions.

Haff's solutions have produced results, but they have several limitations. While there are no adjustable parameters to contend with, physical systems of practical interest do not fall within the range of applications of these equations. Still, these ideas have found applications in many problems, such as the sounding mechanism of booming dunes and squeaking sands. In addition, they further contribute to a fundamental basis for understanding granular flow in a fluid mechanical sense.

It is also useful here to define several parameters related to granular flow that will arise in the coming discussion of booming mechanisms. These parameters also help to explain what it means for a granular material to be fluidized. Bagnold, a prominent early investigator of granular physics, starts by assuming that the grains are spherical and defining a linear concentration parameter, $\lambda = D/s$, where D is the average diameter, and s is the average separation distance between grains. Three critical values of λ can be seen experimentally which correspond to the three major regimes of behavior which granular matter can exhibit: compacted and stationary; mobile but still intimately in contact; and highly energized with only occasional collisions between grains.[12] When λ approaches ∞, the grains are all in contact and the material effectively behaves as a solid. In the case that $\lambda <\sim 17$ the granular material behaves as non-Newtonian fluid. That is to say, it flows but exhibits resistance to shear even with zero shears applied, like a thick mixture of cornstarch and water.[13] When $\lambda <\sim 14$ the grains are sufficiently fluidized to behave as a Newtonian fluid, such as water, which does not resist shearing until it is already in motion. The average flow rate of the material is often referred to as u. In the case of avalanching material, in which only the top layers of the material are moving, the difference in velocity between the layers of moving grains is denoted by Δu. Because of the discrete nature of granular materials most grains travel at slightly different velocities, prompting researchers to describe the average difference between the velocities of individual grains from the average with the variable $\bar{v}$.

G. Size segregation

The final unusual property of granular material covered here is size segregation, a phenomenon, which seems to defy thermodynamic laws and defy physicists' attempts at elucidation. Observing a can of mixed nuts that has been shaken makes the effect readily observable. The larger Brazil nuts will have migrated to the top of the container and the smaller peanuts and fragments will generally have moved to the bottom of the can. In reference to this simple example, this phenomenon is often referred to as the "Brazil nut effect". In most situations, a large intruder particle in a monodisperse granular aggregate of smaller particles will move to the top of a container when the container is vibrated. A number of the major attempts to describe this phenomenon, convection theories, percolation theories, and a hydrodynamic theory, are all discussed here, qualitatively due to their complexity and length constraints. Both of the older mechanical theories and the new fluid dynamic theory are discussed by Trujillo and Herrmann.[14]

As a result of frictional forces, the particles in the center of a vibrated column of granular material tend to rise toward the top while the particles near the walls of the container tend toward the bottom, and it is thought that this type of mechanism carries larger intruders to the top of the container. In this situation, it is decompaction rather than fluidization that we are discussing. The particles remain in contact but can change their configuration due to energy imparted by the vibration of the container. Coefficients of friction between particles and between the particles and the walls are taken to be different, and it is friction with the walls that prevents grains along the walls from moving upwards. This leads to convection rolls at the edges of the container. Vibrations move all grains, large and small, that are closer to the center of the container up towards the top, where the smaller grains are carried back down again by convection rolls, leaving the larger intruders stranded at the top of the container.

Percolation models take a slightly different tack, suggesting, instead, that it is a general downward movement of smaller particles that leads to the rising of larger intruders. In this model, smaller grains can easily fall into voids opened beneath them by periodic vibration, while the intruders can only fall into larger voids. This filling in of voids beneath intruders is what effectively causes their rise. Larger particles can also contribute to the creation of voids beneath themselves by encouraging formation of arches. As the smaller particles fill the voids beneath the larger intruders, the large intruders gradually move upwards as the net downward movement of smaller grains displaces them.

Trujillo and Herrmann take a kinetic, hydrodynamical approach to the problem. It is first assumed that all particles are made of material of the same density, but this still allows different regions of the material to have different densities when considered on a length scale significantly larger than an individual particle. Density variations lead to a difference in kinetic energy between regions of different densities; where the energy variations arise from the dissipative nature of the collisions between particles. The intruder experiences an effective buoyant force. The authors derive a continuum formulation for the granular fluid, introduce a granular temperature, and propose an analytic method for estimating the local temperature. The also take into account other effects such as a thermal expansion, really just a change in pressure, due to changes in granular temperature and dependence of the phenomenon on particle size.

This phenomenon is more complex than any of the models described above, and has enormous practical consequences. For instance, segregation speed depends on the density of the intruder in some cases, but this dependence vanishes as the interstitial air is removed from the container.[15] In a related case, the axial rotation of a cylinder filled with different sized materials will cause grains of different sizes to segregate into alternating, transient band patterns (FIG. 13), whose characteristics depend on the speed of rotation.

Segregation of this type is undesirable when mixing of two differently sized materials must be affected, for instance in the precision manufacture of pharmaceuticals. Without an understanding of the underlying physics, methods of effectively mixing powders and grains must be found by trial and error. The problem itself and the theories describing it are far more complex and nuanced than the simple ideas introduced here, but it should be clear thus far that this issue is a complex one, just as granular physics as a whole is not well understood.

FIG. 13. Bands in white table salt and fine, black sand that spontaneously formed when the cylinder was continuously rotated around its axis.[16]

III. BOOMING AND SQEAKING SAND

Some speculations as to the cause of the acoustic emissions of booming dunes are over 1500 years old, and, despite modern scientific research in the last centuries, even ideas developed recently remain speculation. Scientific investigations have lumped booming avalanches together with other types of acoustically active sands on the assumption that the mechanism that causes the emissions in all of these sand types is the same or at least related. In an attempt to understand the underlying physics researchers have extensively studied the emissions of booming dunes and squeaking sands, as well as the properties of the grains themselves, producing a good deal of information but little in the way of answers.[5,17-19] Theoretical investigations are few and far between, only a handful exists, but currently available models are coming closer to explaining the elusive mechanism. [2,12] The remainder of this section describes the emissions and properties of acoustically active sand in more detail and reviews the two most recent mechanism theories.

A. Acoustic emissions of booming and squeaking sand

I pause here briefly to remind the reader of the nature of sound before discussing the sounds produced by booming and squeaking sands. Sound propagates through the vibration of air molecules as a longitudinal pressure wave. The vibrations are caused when the vibration of a physical object immersed in air causes a corresponding vibration in the surrounding air. This means that, in a search for a sound producing mechanism, we are looking for a mechanism that causes a sustained, coherent vibration in the acoustically active sand.

The actual acoustic emissions from booming and squeaking sand are distinct from each other in type and vary widely within these types from place to and with ambient conditions. The dominant frequencies of booming avalanches range from 50-200 Hz at different locations at different times and are accompanied by several other frequencies at lower amplitudes. Booming events also demonstrate a distinct beat or interruption frequency of approximately 1 Hz and generate accompanying seismic vibrations. Squeaking sands produce much higher and purer tones when struck or sheared that consist almost entirely of a single dominant frequency that ranges from 500-1500 Hz, depending upon location of the sand and conditions.[2]

B. Properties of Booming and Squeaking Grains

Because not all sands boom or squeak, it is assumed that some physical characteristics distinguish booming and squeaking sand from normal sand; thus I introduce several of these characteristics here. One of the simplest analyses of sand is the determination of size sorting. The mean grain size of nearly all sands, booming and silent, is ~300 μm.[1] By sieving sand to separate out the various sized grains, it is possible to determine the percentage of the sand that is made up of grains of a certain size. Grains shapes also have varying sphericities, a measure of how far the shape deviates from a perfect sphere, and are variably well-rounded, meaning that the grains lack abrupt surface asperities. The surface textures of grains, often as observed under a Scanning Electron Microscope (SEM) on the μm scale, are also characteristic. Several SEM micrographs of normal, squeaking, and booming sand are shown in FIG. 14. A final property of interest is the shear resistances of these sands.

Booming and squeaking sands are comparable to one another as measured by some, but not all, of these characteristics, and both sometimes exhibit properties similar to silent sand. Squeaking sand is well sorted, well rounded, highly spherical, smooth, and demonstrates a high resistance to shearing, whereas highly smooth, polished grains distinguish booming sand more than anything.[1] With the exception of the booming back beach dunes on the island of Kauai in Hawaii, whose grains are calcium carbonate,[19] all booming and squeaking sands are quartz. Though some properties do tend to distinguish acoustically active sand from normal sand, none of the properties point to an obvious sounding mechanism for either booming or squeaking sand.

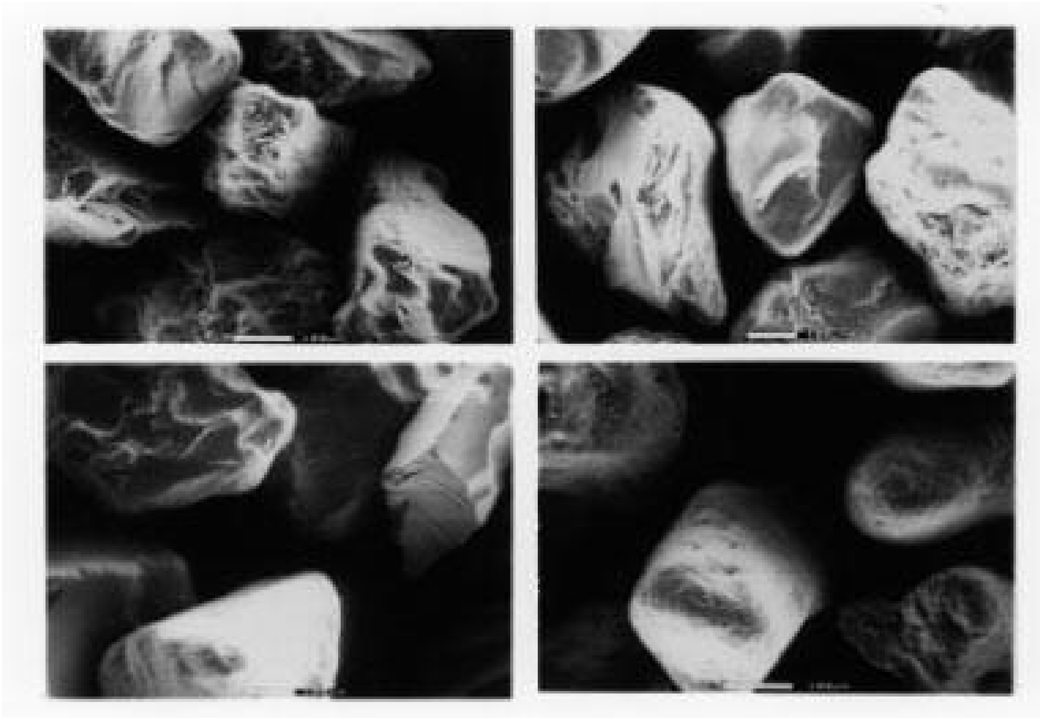


FIG. 14**.** A composite diagram (from top left to right) of normal beach (top left), squeaking beach (top right) and booming desert (bottom) sand grains using low-magnification electron microscopy. Samples were collected from Lake Huron at Bay City, MI (top left), Lake Michigan at Ludington, MI (top right) and Sand Mountain, NV (bottom). The sample in the bottom right panel was sieved and consists of grains smaller than ~200 μm. All micrographs were made on the 100 μm length scale. These photos suggest that the normal beach sand is poorly polished and irregular in shape, while the squeaking sand is more polished. Occasional scour marks appear on both types of beach sands, but not on booming sand. While squeaking grains are by and large rounded, booming sand contains a variety of erosional grain states as shown in the bottom left panel, including many smaller, well-polished, well-rounded grains, as seen in the bottom right micrograph. The top-right grain in the bottom left panel is highly unusual in booming sand.[1]

There is also a disparity between the reactions of these types of sands to moisture and other environmental factors. Booming only occurs under very dry conditions and as little as a few drops of water in a one liter bag of booming sand can preclude booming. Squeaking sand performs most readily when dry, but it will still squeak weakly in some cases when totally covered by water. Both types are more active when hot, but this may be due to lower moisture.[1]

C. Major experimental results and their effects on theory

The majority of the work published on booming and squeaking sand is experimental in nature, probably due to the general lack of a theoretical basis for granular physics, and it has succeeded in eliminating several factors from consideration. The size distribution of the sand, high sphereicity of the grains, piezoelectric effects, and vibrating air pockets are among the now discounted factors.

The size and well-sorted nature of most squeaking sand were thought to be important for booming, but experimental work has shown that a particular grain size or particular size distribution is not sufficient condition for the onset of booming. Haff separated all of the grains within certain size ranges out of a sample of booming sand and found that all sizes of booming grains still boomed when isolated.[5] Also, Lindsay et. al found that booming sand from Sand Mountain in Nevada was just as sorted as nearby sand from silent dunes.[18] It has been shown that monodisperse glass spheres do not boom, meaning that well-sorted-ness is not sufficient to produce booming. This last example also shows that sphereicity is insufficient for producing booming.

Another once-favored avenue of research was the pursuit of effects from the piezoelectric properties of quartz. The reaction of booming sand to even small amounts of water, which would have interfered with the charges on the surfaces of the sand grains, and the fact that most booming sand is quartz initiated this curiosity. Piezoelectric crystals such as quartz produce a voltage when they are strained or stressed and, correspondingly, change shape in response to an applied voltage. In some experiments electrically grounded sand still produced acoustic emissions, failing to support this theory. Also contradicting this hypothesis are the booming beach dunes of Hawaii, whose sands are made of calcium carbonate.

Vibrating air pockets left after the evaporation of water from interstitial spaces in booming dunes were also once thought responsible for booming. Experiments performed by placing booming sand samples in a Bell jar and removing air have shown that the sound producing vibrations are still present without the air. While the experiments discussed in this subsection have failed to discover why some sands boom and others do not, they have helped to direct theorists in the right directions.

C. Bagnold's theory of the booming mechanism

R. A. Bagnold, the great English geologist and engineer, wrote the seminal work on the physics of dunes.[20] He also studied booming and squeaking sand to some extent and believed the vociferousness of both sands to result from the same mechanism.[12] The semi-empirical theory that he developed to explain it can, given mean grain size, predict a single frequency of emission of a booming event. The theory is incomplete and does not account for the multiple frequencies produced by booming avalanches, but it does serve as a good starting point. A derivation of this theory appears in the appendix. A more qualitative look at the theory comes in the following material.

Consider a single plate of sand of some depth sliding down the face of a dune with a distinct slip plane between the moving and stationary sand (FIG. 15a). Imagine that the avalanching grains are moving collectively with a net velocity straight downhill with a common, average flow velocity *u*, with all other velocities of individual grains averaging to zero. As this coherent chunk of sand slides down the slip plane, the grains on the bottom of the moving chunk bounce along and collide with the stationary grains, rebounding and imparting momentum normal to the slip plane to the rest of the moving chunk. This results in a repulsive pressure P between the moving layer and the stationary layer, which is proportional to the velocity. At the same time the moving layer experiences a downward force Q due to gravity.

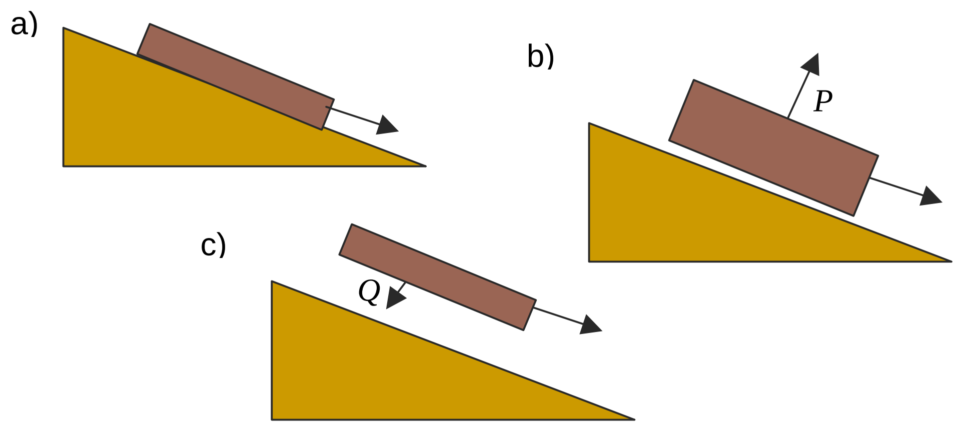


FIG. 15. Qualitative depiction of Bagnold's Theory. a) the sand approaches the terminal velocity U_c and b) dilates as P overwhelms Q momentarily, after which the sand c) compacts back down to the surface of the slip plane under Q so that it can again gain velocity and approach U_c.

As shown in the appendix, Bagnold derived a terminal velocity U_c, inversely proportional to λ. If the moving sand exceeds U_c for a moment, λ must decrease from its starting point of 17, which makes P momentarily larger than Q. This causes the moving sand to dilate higher above the surface of the dune. The density decrease also decreases P, causing the expanded sand to rapidly compact back down to the surface of the dune under its weight. Again, λ increases and the cycle repeats as the sand crosses U_c. The frequency with which the layer of moving sand alternately expands above and crashes onto the stationary surface of sand beneath is the frequency of the booming emission. Bagnold derived the following formula to model the supposed mechanism:

$$f = \sqrt{\frac{g\lambda_{\min}}{8D}}, \qquad (10)$$

where f is the frequency of sound and λ_{min} is the minimum λ for which the moving grains should just barely be passing over the stationary grains, in this case, $\lambda_{\min} \approx 14$. If we do an order of magnitude calculation with $D \approx 300\mu\text{m}$, we find $f \approx 240\text{Hz}$. The model can also be applied to squeaking sand. Impacting squeaking sand also causes shearing and the formation of slip planes and leads to dispersive stress. By multiplying by a constant $\sqrt{K_0}$ in equation (10) above, Bagnold proposed to account for the fact that striking or compressing subjects the moving grains to an acceleration K_0 times larger than g. This constant varies from material to material.

Bagnold's model's predicted frequency is outside the ranges of a number of booming sands. The model also cannot account for the beats and other tones heard during booming events. On the other hand, the emissions of squeaking sand fit this model's prediction better, in that the sound they produce is closer to a single dominant frequency.

D. Fluid mechanical theory of the booming mechanism

In a more recent paper, A. J. Patitsas uses fluid-dynamical methods, modified for use with granular media, to predict the existence of slip channels, in which mechanical energy is converted into sound waves to produce the booming emission of booming sand.[2] The equations of motion can be worked out such that only the coefficients of friction and restitution for the sand and the ratio of flow velocity to random velocity $(\bar{v}/\Delta u)$ are needed to determine the behavior of the avalanching sand. The coefficients of friction and restitution are particularly affected by the surface texture of the grains and the existence of water on the grains, which suggests that it is the surface texture of the sand grains that leads to booming.

Imagine that Bagnold's single block of sand sliding down the surface of a dune actually has several slip channels and a downhill velocity gradient that varies with the depth in the direction normal to the surface of the dune. In other words, the sub layers at the top of the moving layer are moving downhill faster than the deeper layers, and between these layers are slip planes parallel to the dune surface of distinct width in which material is moving upward normal to the plane. Excitation of elastic modes of vibration in these slip layers causes the conversion of energy into acoustic emissions. This theory predicts the location and number of slip layers, which each correspond to a different frequency corresponding to their width.

An experiment performed by Shiego Miwa, a retired Japanese researcher with an interest in musical sand, synchronized the striking of a bed of sand with the taking of an x ray image of the sand. Less dense, fluidized regions of sand show up as lighter regions in the x rays. It can be seen (FIG. 16) that the silent sand responds little to striking while the squeaking sand exhibits slip channels (the light bands emanating from the rod in the x ray) during an impact from the rod. This is encouraging news for this theory.

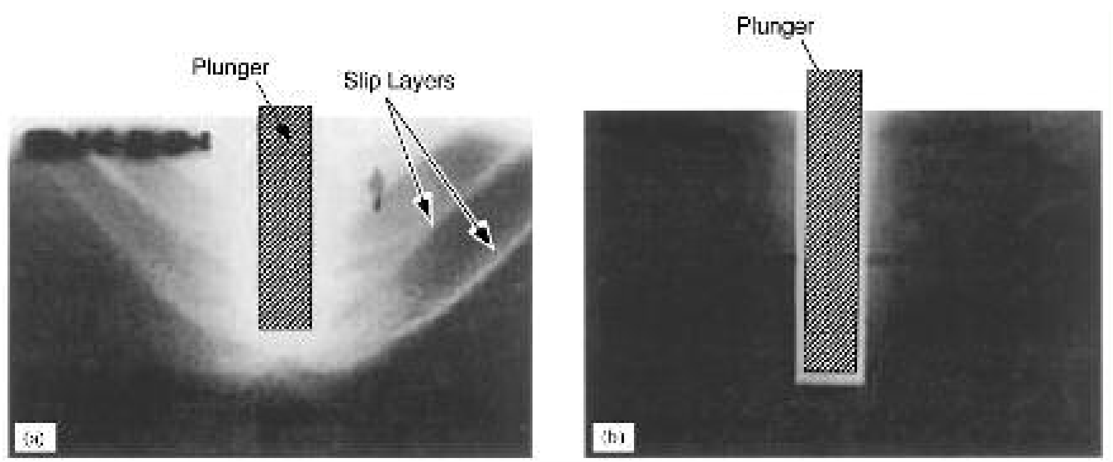


FIG. 16. X-ray images taken by Shiego Miwa of silent and squeaking respectively as a rod impacts them. The slip channels are apparent in the acoustically active sand.[2]

Because this fluid mechanical theory can predict the frequencies that should be present in the acoustic emissions of sand for which the necessary physical properties are known it is possible to compare the results of this theory with experiment. Below in FIG. 17 and FIG. 18, two recordings of the emissions of a sample of frog sand, a relative of squeaking sand, are shown, as well as their corresponding Fourier transforms in FIG. 19 and FIG. 20. Table (2) lists, side by side, a number of the frequencies recorded in both events as well as the frequencies that theory predicts should be present. Though this theory is incomplete, the correspondence between the expected and measured frequencies is encouraging.

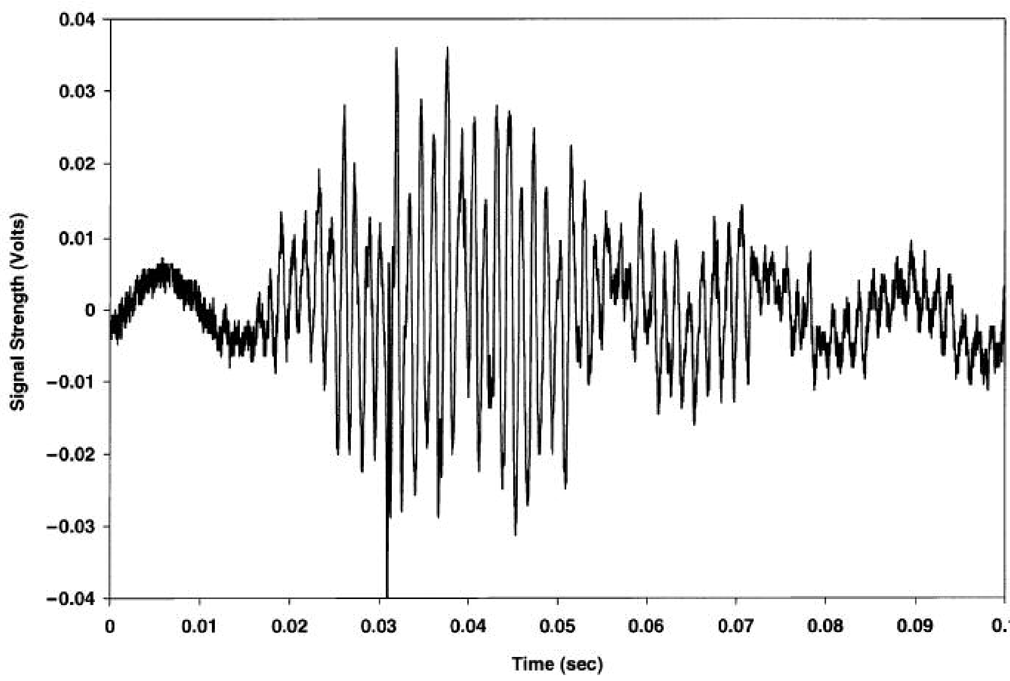


FIG. 17. Miwa's first recording of Frog Sand emissions.

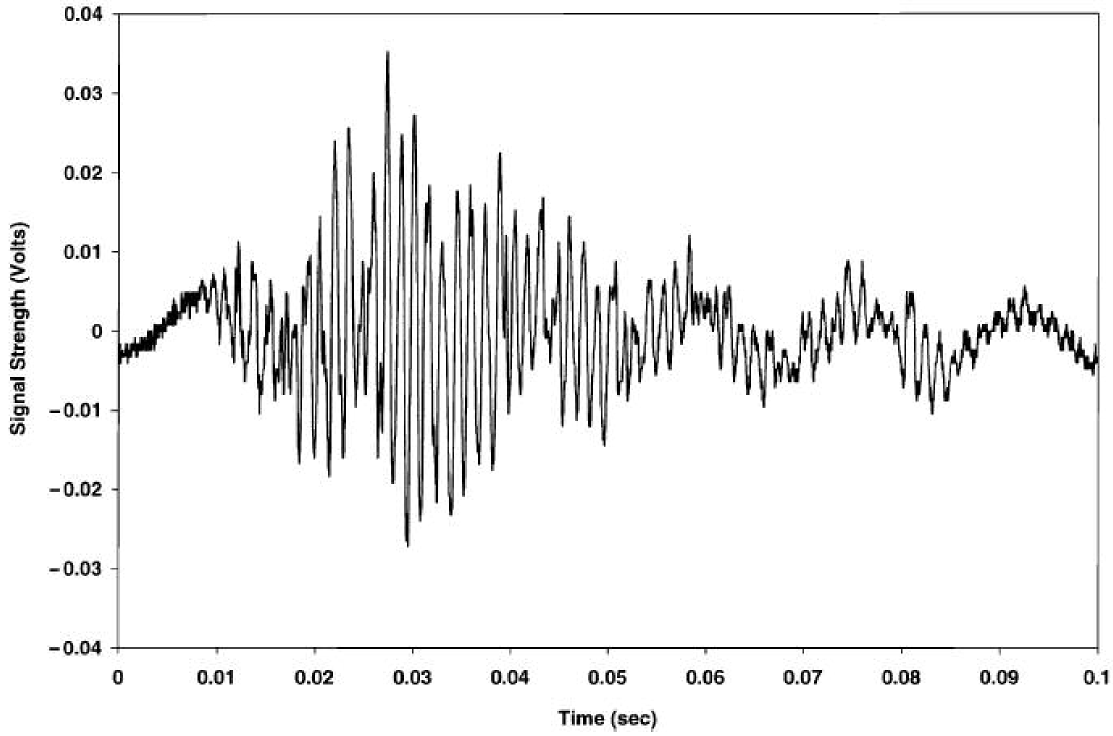


FIG. 18. Same as FIG. 17. but taken one month later

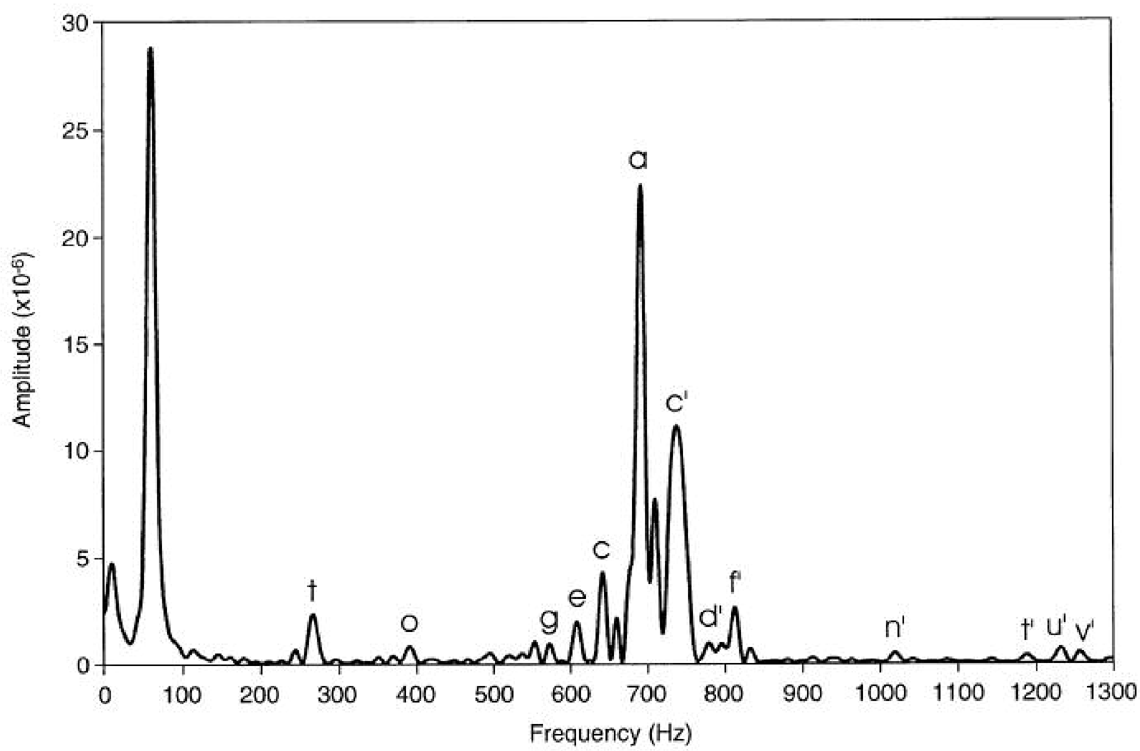


FIG. 19. Fourier transform of signal in FIG. 17. 60Hz peak visible.

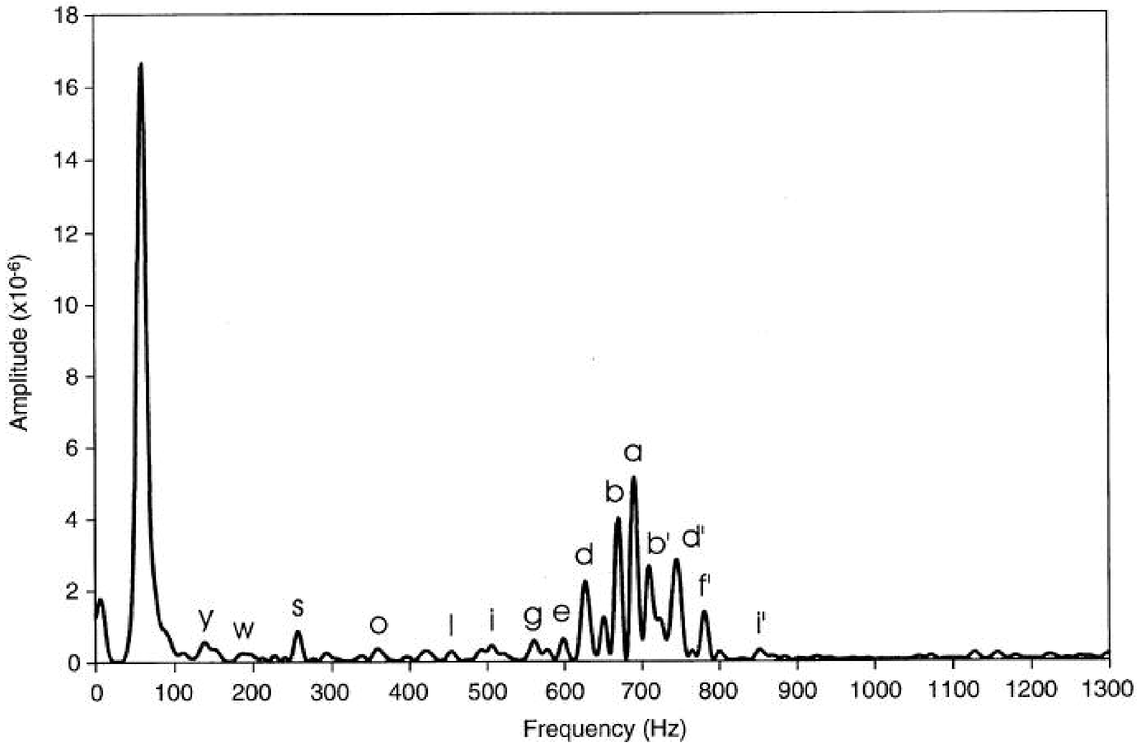


FIG. 20. Same as FIG. 19, except corresponding to Fig. 18.

Frequency spectra for the frog sand

Peak	Frequency(Hz) Fig. 8	Frequency(Hz) Fig. 9	Frequency(Hz) theoretical
y		139	
w		191	
t	266		
s		258	
o	390	360	
m		422	
l		455	496
j		494	513
i		506	536
h	553		552
g	572	560	571
f		577	593
e	607	598	608
d		628	630
c	641	650	649
b	660	670	665
a	690	689	688
b'	708	709	705
c'	737	721	723
d'	779	744	745
e'		762	761
f'	811	779	781
g'	833	798	802
i'		850	817
n'	1019		839
t'	1187		858

Table 2. Partial listing of frequencies presenting frog sand emissions from both recordings in the first two columns of numbers. The third column shows theoretical values.

IV. CONCLUSION

We have seen that granular physics is a very complex but very important field of study, and that many questions still lack satisfactory answers, like the mystery of the acoustic emissions of booming dunes. Bagnold's model, based on the mechanics of sand flow and shearing does predict reasonable values for the frequencies of some dunes, but it cannot explain the beat frequency or why so many booming dunes' frequencies cannot be predicted by it. The more recent fluid-dynamical approach seems to be more successful. It predicts the existence of multiple slip channels, which have been observed experimentally, and it can predict a range of frequencies, which have also been observed experimentally and which agree somewhat with the theory to some extent. More work is needed to further elucidate the booming mechanism and to determine what makes some dunes boom while others are silent.

Acknowledgments

Thanks to Carleton College, Bill Titus, Bruce Thomas, and all of my other professors. Also, thanks to my fellow students who have made it so much phun to be a physics major.

Appendix: Derivation of Bagnold's Model

Bagnold's model of the sounding mechanism of booming sand begins with an experimental and theoretical study of the terminal velocity of a sheet of sand flowing down the face of a dune. To measure the intergranular stresses during shearing, he sheared hard spheres with a density σ in a liquid of density ρ, which balanced σ to simulate a gravity free environment, and extrapolated the results to model the less ideal dynamics of a dry sand avalanche. The Bagnold model for the sounding mechanism is based on a derivation of a terminal velocity for the moving front of a sand flow and an analysis of the stresses on a thin sheet of sand sliding down a dune face. The details of the model are derived below. In addition, they are given in Bagnold's original paper and are also shown nicely in Sholtz, et al.[1,12]

Imagine a collection of uniform spheres of diameter *D*, and recall Bagnold's linear concentration parameter $\lambda = D/s$, where *s* is the mean intergranular separation. For close-packed, equal sized spheres, the volume fraction *C* is $C_0 = \pi/3\sqrt{2} \approx 0.74$. If the spheres are dispersed and mean distance between adjacent *centers* becomes *bD*, where $b > 1$, *s* becomes larger than the *s* measured for close-packed spheres and can be related to *b*, as shown here

$$b = \frac{s}{D} + 1. \tag{A.1}$$

This can be expressed in terms of *C*

$$C = \frac{C_0}{b^3} = \frac{C_0}{\left(\lambda^{-1} + 1\right)^3}. \tag{A.2}$$

It is also important to keep in mind that flow will not occur until $\lambda < 17$.

Bagnold also made three major assumptions about the flow and shear of the spheres. First, the spheres are in a uniform state of shear strain, and the mean relative velocity between the spheres and the interstitial fluid is zero. This can be expressed as $\nabla\left(dU/dy\right) \approx 0$, where *U* is the mean flow velocity in the +x direction. Second, frictional loses maintain a constant kinetic energy per unit volume. And third, the spheres also make small oscillations in all three directions.

It is really shearing of adjacent layers that we need to analyze, and we begin to describe the geometry and motion of the spheres here. We will be considering the slipping of the spheres in plane B over the spheres in plane A, as shown in FIG. A.1, in a series of jumps. The layers of spheres lie in planes parallel to the x-z plane and are stacked upon each other in the y-axis. The x-axis is considered to be pointing down hill. In this model the average differential velocity between planes is $\delta U = kbD\left(dU/dy\right)$, where *k* is a constant varying between $1/\sqrt{2}$ a $\sqrt{2/3}$ depending up on the geometry of the spheres.

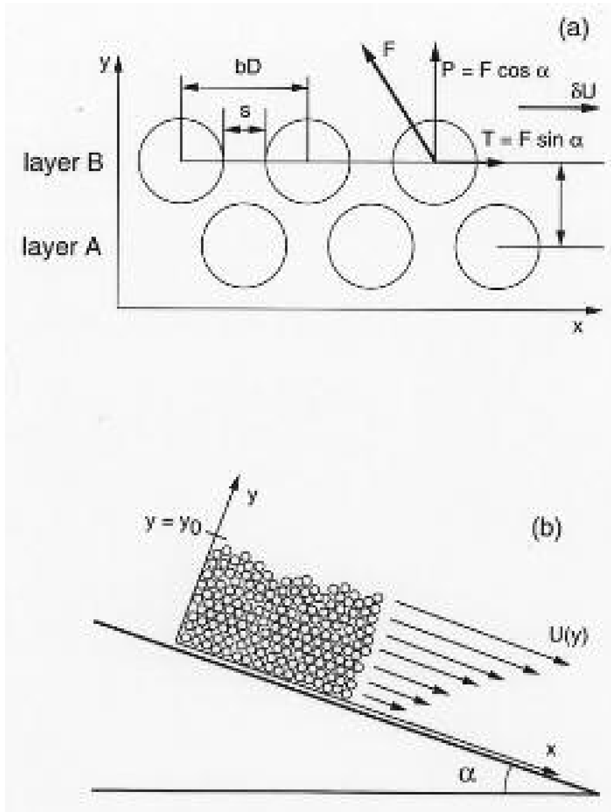


FIG. A.1. Diagram from Sholtz, et al. to accompany the derivation of Bagnold's semi-empirical formula for the frequency of a booming dune.[1]

Now we will examine the mechanics of individual spheres and from this derive expressions for the shears and stresses upon the flowing layers of spheres. First, make the assumption that a single sphere in layer B makes $f(\lambda)\delta U/s$ collisions per unit time with other spheres in layer A. There are $(bD)^{-2}$ spheres in a unit area of the plane of layer B and each collision of a sphere will change the sphere's momentum by $2m\delta U\cos\beta$, where m is the mass of the sphere and β is determined by the collision conditions. Bringing these quantities together, it can be seen that layer B should experience a net pressure P_y along the y-axis

$$P_y = (bD)^{-2}\frac{f(\lambda)\delta U}{s}2m\delta U\cos\beta, \qquad \text{(A.3)}$$

or

$$P_y = r\sigma\lambda f(\lambda) D^2 \left(\frac{dU}{dy}\right)^2 \cos\beta, \quad \text{(A.4)}$$

where

$$r = \frac{2mk^2}{\sigma D^3}. \quad \text{(A.5)}$$

There is also a tangential shear stress T_{xy} as shown here,

$$T_{xy} = P_y \tan\beta. \quad \text{(A.6)}$$

At this point, we turn to experimental work to validate and complete this theory. Bagnold's experiments with beads in a "gravity free environment" found T_{xy} and P_y proportional to $(dU/dy)^2$, in agreement with theory. Experimental work also gives us reasonable values for the two remaining unknown values, $f(\lambda)$ and β, and tells us that when $\lambda < 12$ the behavior of the aggregate of spheres is close to the behavior of a Newtonian fluid. The relevant quantities are $f(\lambda) \approx \lambda$, $\tan\beta \approx 0.32$, and $r = 0.042$, which leads to the numerical relation below for $\lambda < 12$,

$$P = 0.042\sigma\lambda^2 D^2 \left(\frac{dU}{dy}\right)^2 \cos\beta. \quad \text{(A.7)}$$

Though not it is not an obvious assumption, we will assume that these relations hold when irregularly shaped sand grains are sheared in air.

Now we will use these relations to examine the shear stresses on a single plane of moving grains below the surface of the flowing layer on the face of a sand pile. The applied shear stress is

$$T_{xy} = \sigma g \sin\alpha \int_y^0 C(y')dy', \quad \text{(A.8)}$$

where α is the angle of incline of the face of the sand pile and, again, $C(y)$ is the previously found equation () for the volume fraction. Equating the previous two equations, because air is not very viscous, and solving for (dU/dy),we arrive at

$$\frac{dU}{dy} = \left(\frac{g\sin\alpha}{r\sin\beta}\right)^{1/2} \frac{\left[\int_y^0 C(y')dy'\right]^{1/2}}{\lambda D}. \tag{A.9}$$

We then make the assumption that C is roughly uniform throughout the depth and take Bagnold's empirical result of $C = 0.6$. At the shear plane, the sand will be just decompacted enough to flow, meaning that $\lambda = 17$. Using this value and taking $r\sin\beta = 0.076$ we can reduce the above expression down to

$$\frac{dU}{dy} = 0.165(g\sin\alpha)^{1/2}\frac{y^{1/2}}{D}, \tag{A.10}$$

which can be integrated to

$$U = \frac{2}{3}(0.165)(g\sin\alpha)^{1/2}\frac{y_0^{1/2}}{D}, \tag{A.11}$$

where y_0 is the height of the flowing layer.

With a few more assumptions about the flow, equation (above) can be further simplified. Take the interfacial velocity at any shear surface to be $D(dU/dy)$, and simplify the expression for pressure to

$$P = r\beta\sigma\lambda^2 U^2 \cos\beta. \tag{A.12}$$

For slow continual shear, we can take the value $\lambda \approx 17$ for the local linear concentration and find

$$U_c = \frac{1}{17}\sqrt{\frac{P}{r\sigma\cos\beta}}. \tag{A.13}$$

which is the critical velocity. Bagnold performed experiments with Bulldozed sand to verify that these results hold for shearing, avalanching sand.

With these results in hand, we can apply them to booming sand. The mass of moving sand exerts a compressive stress Q on the shear surface. If the system is in equilibrium, increasing U above U_c requires a dilation that will decrease λ below 17, momentarily causing P to exceed Q, accelerating the moving layer of the sand slightly upward. But as the sand dilates, P decreases rapidly and the sand collapses back down. The recompacted sand again exceeds the terminal velocity and dilates, repeating the cycle. The oscillating force in the normal direction is $mg - P$, where m is the mass of the sand. The minimum mean local dilatation at the shear surface at which oscillations can still occur is $D/\lambda_{\min}$. The stress will only be effective when λ is near this minimum because at this value the planes will barely clear one another and because P varies rapidly with λ. The rise and fall of the overburden through the distance $D/\lambda_{\min}$ will be in almost free-fall, meaning that the minimum frequency of oscillations is given by

$$f = \sqrt{\frac{g\lambda_{\min}}{8D}}. \qquad \text{(A.14)}$$

References

[1] P. Sholtz, M. Bretz, and F. Nori, Contemporary Physics **38** (5), 329 (1997).
[2] A. J. Patitsas, Journal of Fluids and Structures **17** (2), 287 (2003).
[3] Ivan Amato, Science **254** (5034), 938 (1991).
[4] H. L. Fairchild, Science **51** (1307), 62 (1920).
[5] P. K. Haff, American Scientist **74** (4), 376 (1986).
[6] Jacques Duran, in *Sands, powders, and grains : an introduction to the physics of granular materials* (Springer, New York, 2000).
[7] Heinrich M. Jaeger, Sidney R. Nagel, and Robert P. Behringer, Physics Today **49** (4), 32 (1996).
[8] Eugene Hecht, *Optics*, 4th ed. (Addison Wesley, San Francisco, 2002).

9 H. M. Jaeger and S. R. Nagel, Science **255** (5051), 1523 (1992).
10 Robert Paul Behringer, Behringer's Nonlinear Flow Group Page, <http://www.phy.duke.edu/research/ltb/ltbgroup.html>,accessed 12 March 2004, (2004), Vol. 2004.
11 P. K. Haff, Journal of Fluid Mechanics **134** (SEP), 401 (1983).
12 R. A. Bagnold, Proceedings of the Royal Society of London. Series A **295** (1442), 219 (1966).
13 T.E. Faber, *Fluid dynamics for physicists*. (Cambridge University Press, New York, 1995).
14 Leonardo and Hans J. Herrmann Trujillo, Physica A **330**, 519 (2003).
15 Peter Weiss, Science News **160** (20) (2001).
16 Peter Weiss, Science News **164** (4) (2003).
17 H. Takahara, 1973 (unpublished); J. Qu, B. Sun, W. M. Zhang et al., Chinese Science Bulletin **40** (20), 1719 (1995); K. Nishiyama and S. Mori, 1982 (unpublished); M. F. Leach, D. E. Goldsack, and H. J. Chartrand, presented at the 1995 IEEE Ultrasonics Symposium. Proceedings. An International Symposium. vol.1 7-10 Nov. 1995 Seattle, WA, USA [IEEE Ultrasonics, Ferroelectr. & Frequency Control Soc], 1995 (unpublished); Z. M. Chai, M. F. Leach, and G. A. Rubin, (unpublished); D. R. Criswell and J. F. Lindsay, presented at the 54th Annual Meeting of the American Geophysical Union. (Abstracts only). 16-20 April 1973 Washington, DC, USA [American Geophys. Union], 1973 (unpublished).
18 J. F. Lindsay, D. R. Criswell, T. L. Criswell et al., Geological Society of America Bulletin **87** (3), 463 (1976).
19 D. R. Criswell, J. F. Lindsay, and D. L. Reasoner, Journal of Geophysical Research **80** (35), 4963 (1975).
20 Ralph A. Bagnold, *The physics of blown sand and desert dunes*. (W. Morrow & company, New York, 1942).

Annotated Bibliography

Challenges in granular physics. (World Scientific, Singapore; New Jersey, 2002).

This book is yet another collection of conference proceedings, edited again by Anita Mehta. Like the other book she edited, this one can give a very broad overview of granular physics if read lightly but is also packed full of complicated pieces for the reader who wants more. Again, this book was frequently over my head, and there was no mention of booming sand.

Jacques Duran, in *Sands, powders, and grains : an introduction to the physics of granular materials* (Springer, New York, 2000).

This book is a very helpful introductory text on granular physics for advanced undergraduates and graduate students. It also happens to be the only introductory text on granular physics in print. Duran covers some background, grain-grain

interactions, bulk static behavior, bulk flow behavior, mixing and segregation, and numerical methods including cellular automata models. I have adapted many of the explanations from this book. There was no mention of booming sand.

Granular matter : an interdisciplinary approach. (Springer-Verlag, New York, 1994).

Like many of the other books on granular physics, this is a collection of papers and conference submissions covering very specific topics that were more complicated than I would have liked them to have been. Still, I learned something about the history of granular physics from the introduction, and skimming through the abstracts and introductions gives one a nice sense of how broad this field is. Again, there was no mention of booming sand.

P. K. Haff, American Scientist **74** (4), 376 (1986).

Studying booming sand must be what granular physicists do on weekends for fun. That is the impression I got from reading this paper. The experiments described here are important, basic, and creative; and there is little math or abstruse theory. It is written on a more popular level.

P. K. Haff, Journal of Fluid Mechanics **134** (SEP), 401 (1983).

This is a very important and heavily cited foundation for modern attempts to describe granular physics by way of modifying theories from fluid dynamics. Haff finds fluid dynamic equations of motion, which can be solved analytically and can be applied to granular materials.

H. M. Jaeger and S. R. Nagel, Science **255** (5051), 1523 (1992).

This article is a good overview of many of the concerns that granular physicists are attempting to deal with.

A. J. Patitsas, Journal of Fluids and Structures **17** (2), 287 (2003).

Patitsas's paper is very dense. The entire second page and a half of the third page of the paper are devoted to a table (in small type) of the variables he uses in his mathematics. Still, it does summarize some of the same basic information as Sholtz et. al. and gives qualitative descriptions of the theory as well, which do shed some light on the matter. This is the most recent, and, so far, most successful attempt to discover the secrets of booming dunes. It is worth reading through, and some of the ideas presented about the possible presence of similar phenomenon on the moon and on Mars are very exciting.

P. Sholtz, M. Bretz, and F. Nori, Contemporary Physics **38** (5), 329 (1997).

I spent a lot of time reading and re-reading this paper. It is the most comprehensive review article of research on the physics of booming dunes up to 1997. It covers everyone from J. J. Thompson and others in the last part of the 19th and early twentieth centuries, through Bagnold's theory and Miwa's in situ x ray imaging of acoustically active sand. This is a good place to start learning about booming sand.

Leonardo Trujillo and Hans J. Herrmann, Physica A **330**, 519 (2003).

This is a very recent paper on hydrodynamic models for size segregation. It also provides a quick overview of other theories of the mechanism that causes size segregation.