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LIGO and the Search for Gravitational Waves

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By Adam Libson

Abstract:

This paper discusses the Laser Interferometer Gravitational-Wave Observatory (LIGO). It is hoped that with these detectors, it will be possible to detect gravitational waves from distant astronomical sources. Though gravitational radiation has been predicted for most of the last century, it has never been directly observed. This is the goal of the LIGO detectors. There are currently two interferometer sites in the United States, one in Hanford, Washington, and the other in Livingston, Louisiana, with other interferometers under construction around the world. Each LIGO detector is a long baseline Michelson interferometer with Fabry-Perot resonant cavities in the arms. This paper addresses the general relativity needed to talk about gravity waves, though I restrict myself to the linear approximation for simplicity. I also discuss how an idealized passing gravity wave will affect the LIGO detectors, along with the physics of the Michelson interferometer with Fabry-Perot cavities. Finally, I look at some of the noise sources in the interferometers, and for a few of them address possible ways around this noise. In closing, this paper will discuss the current research, which, itis hoped, will aid in the detection of gravitational radiation.

Introduction and History

The Laser Interferometer Gravitational-Wave Observatory (LIGO) detectors are part of a world-wide effort to directly detect gravitational radiation. This form of radiation was first predicted by Einstein when he published his revolutionary paper on special relativity in 1905, and the specific laws governing gravity waves were laid out in Einstein's theory of gravity and space-time which he published in 1918. While gravitational waves have been predicted by these theories for almost a full century, they have never been directly observed. There have been several attempts to see gravity waves over the last few decades, but all have failed to produce detections.

While gravity waves have never been seen, there is little doubt in the physics community of their existence. One of the reasons for this is gravitational radiation's solid theoretical grounding. Gravity waves are required by Einstein's theory of relativity, which has accumulated numerous experimental confirmations since its postulation. Newtonian gravity does not provide for gravitational radiation and thus changes in the gravitational field must be transmitted at infinite speed. This violates the theory of special relativity, since under that theory information can be transmitted no faster than the speed of light (c). Since special relativity has been confirmed so thoroughly by experiment, it is generally accepted that gravity transmits information at the speed of light or less. The method of this transmission was predicted by Einstein's theory of gravitation.

The theory of general relativity, formulated by Einstein in 1918, predicted that changes in the gravitational field would be transmitted as waves in space-time. Einstein's theory has successfully predicted many physical phenomena, such as gravitational lensing and gravitational red shifts. Because of these and other experimental confirmations of general relativity, the theory is generally accepted as an accurate description of gravity. Since gravitational radiation is a required element of Einstein's theory, there is a very strong argument for the existence of gravity waves, even though they have never been directly observed.

There is also observational, though circumstantial, evidence for gravitational waves. Joel Weisberg and Joseph Taylor have observed a loss of energy taking place in the orbit of binary pulsar 1913+16 [1]. The amount of energy lost is exactly the quantity that general relativity predicts will be radiated away through gravity waves. Figure (1) shows that the system arrives at its periastron earlier than would be expected if it were not radiating gravity waves. The fact that the system arrives at the periastron early indicates that the orbit is decaying, and it is decaying exactly as general relativity predicts it should. While this does not prove the existence of gravitational waves, it is convincing evidence that they are a feature of our universe.

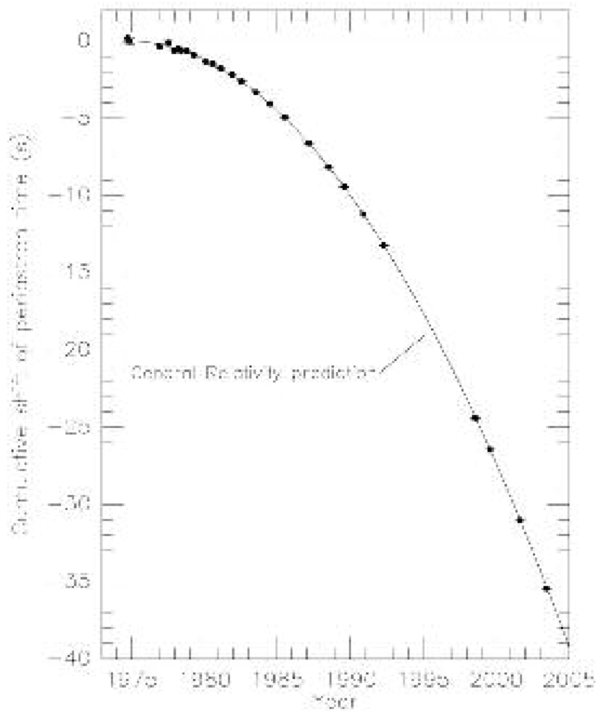

Figure 1: This graph shows the orbital decay of PSR 1913+16 as observed by Taylor and Weisberg. The periastron times are shifted by the decay of the orbit. The line on the graph is the general relativity prediction of the periastron shift time. [Joel Weisberg, J. H. Taylor, *The Relativistic Binary Pulsar B1913+16* (Radio Pulsars, ASP Conference Proceedings, vol. 302, 2003)]

The first devices designed to see gravitational radiation were resonant mass detectors, or bar detectors, and while this paper will not go into the details of their operation, they represent an important step in gravity wave physics. Bar detectors ring like a bell when a gravity wave is passing, and this vibration can be detected using piezo-electric chips or accelerometers. Weber was the first person to build a gravity wave detector and his resonant mass devices started operating in 1966. He claimed to have seen gravity waves with his detectors a few years later, though that claim has since been refuted as the result of faulty data analysis [2]. Figure (2) shows Weber at the University of Maryland with one of his detectors. Several bar detectors are currently in operation around the world, but resonant mass detectors have several major short-comings when compared to interferometric detectors. The mostsignificant flaw is that the detector is only sensitive in a very narrow frequency band. It can only see waves whose frequency is very close to the resonant frequency of the mass being used. While no bar detector has had a convincing detection of gravity waves, these detectors are still valuable tools that continue to contribute a great deal to the science of gravity wave detection.

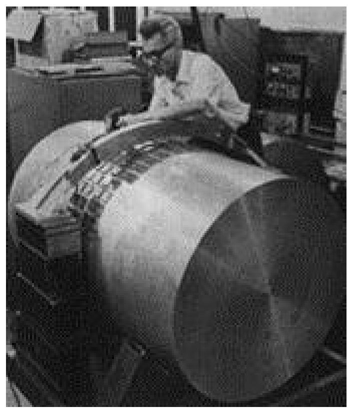

Figure 2: This is a picture of Weber with one of his resonant mass detectors. A passing gravity wave would cause the bar to vibrate, which would then be detected by the piezo-electric chips along the waist of the bar. [Hans C. Ohanian, Remo Ruffini, *Gravitation and Spacetime* 2nd ed. (W. W. Norton and Company, New York, New York, 1994) p. 281]

Following Weber's work, Rainer Weiss investigated the use of interferometers to detect gravity waves in 1972. Using the framework that was developed by Weiss, the first interferometric gravity wave detector was constructed by Forward and Moss in 1972 [3]. Ronald Drever also contributed a great deal to the practice of using interferometers for gravity wave detection. The LIGO interferometers are the descendants of these early experiments. LIGO consists of three interferometers: a 4 km and a 2 km interferometer in Hanford, Washington and a 4 km interferometer in Livingston, Louisiana [4]. The interferometers themselves can be seen in Figure (3). There are several reasons for building multiple interferometers. By using more that one detector, LIGO can have far more confidence in a detection of gravitational radiation. Also, using more than one detector allows for a search for background radiation. Finally, since LIGO is working with a number of detectors around the world, physicists may be able to triangulate the location of an event that produces gravity waves. Eventually, LIGO hopes to participate in gravity wave astronomy and use gravitational radiation to learn more about the universe.


Figure 3: These pictures are of the LIGO interferometers themselves. On the left is the Hanford Observatory which houses a 4 km interferometer, and a 2 km interferometer. On the right is the Livingston Observatory, which has a 4 km interferometer. [http://www.ligo.caltech.edu]

General Relativity and Gravity Waves

The Linear Approximation:

One of the most important concepts in the consideration of gravitation is the calculation of the distance between two points in space-time. In special relativity the distance between two points, ds is described by

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu, \tag{1}$$

where

$$\eta_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{2}$$

is the Minkowski metric of flat space-time[1]. The above tensor is extremely simple partly because of the units used. Throughout the discussion of general relativity, I'll be using units such that $c = 1$. One of Einstein's key ideas in deriving his theory of gravitation was that gravity comes from curvature of space-time. We can describe this curvature using the metric tensor $g_{\mu\nu}$, whose elements will be functions of space and time. This metric, $g_{\mu\nu}$, is just like the Minkowski tensor in that it will allow us to find the distance between two points, except now it will be a distance in curved space-time. The equations of gravitation are then concerned with finding the metric tensor for some region of space-time.

Another problem that must be resolved before we define any equations is the source of the gravitational field. Newton's theory of gravitation has mass as the source of the field; however, this will not work in a relativistic theory of gravity. We might try using energy density, since special relativity says that mass and energy are equivalent, but this will not resolve the relativistic problems. The energy density of a system is not relativistically invariant; it depends on the reference frame of the observer. A static energy density in some reference frame will look like some combination of energy density, energy flux density, and momentum flux density in another frame. As such, the gravitational field measured would depend on the reference frame if energy density were the source. For this reason, the source of the gravitational field must be the energy-

[1] See Appendix A for a discussion of the notation used.

momentum tensor $T^{\mu\nu}$, because this source will define a relativistically correct field [5]. The tensor is defined as follows: T^{00} is the energy density, $T^{0k} = T^{k0}$ is the momentum density in the k direction, and $T^{kl} = T^{lk}$ is the flux of k momentum density in the l direction. This 2nd rank tensor will be the source of the gravitational field. Each term in the tensor will generally be an equation of space and time. While each term of the tensor will not be Lorentz invariant, this tensor may serve as the source of the gravitational field.

We now have the tools we need to look for the equations governing the gravitational field. Since the full non-linear field equations postulated by Einstein in his theory of gravitation are very complex, this paper will not address them. We will, instead, use the linear approximation of Einstein's equations, which approximate weak fields well and are far simpler to use. Since the source of the gravitational field is a tensor of second rank, it is reasonable to assume that the field will be described by a tensor of second rank, which we will call $h^{\mu\nu}$. We also assume that, as for most other forces, the field equation will be a second order differential equation. Finally we require that the field equation be linear, and that it also be symmetric. With these assumptions, we arrive at the linear approximation for the gravitational field equation[2], which is

$$-\kappa T^{\mu\nu} = \partial_\lambda \partial^\lambda h^{\mu\nu} + \partial^\mu \partial^\nu h - \partial_\lambda \partial^\nu h^{\mu\lambda} - \partial_\lambda \partial^\mu h^{\nu\lambda} - \eta^{\mu\nu} \partial_\lambda \partial^\lambda h + \eta^{\mu\nu} \partial_\lambda \partial_\sigma h^{\lambda\sigma}, \tag{3}$$

[2] There is a major problem with the equation for the gravitational field that must be addressed, namely that while matter produces a field, the field produced does not act on matter. The gravitational field as it is formulated in equation 3 will not change the momentum of a particle. This can be corrected by adding a term that corresponds to the Energy and Momentum of the gravitational field to the Energy-Momentum tensor. This term allows the field to transfer energy and momentum from the field to a particle. This process however becomes rather complex quite quickly, and we will thus ignore the problem, since it does not play a large role in the study of gravity waves. For more on this see *Gravitation and Spacetime* by Ohanian and Ruffini, chapter 3.

where h is the trace of the field tensor, and κ is a constant that must be determined by experiment [6]. Each of these terms describes some specific behavior of the field, for example $\partial_\lambda \partial^\lambda h^{\mu\nu}$ gives the wave solution of the field tensor.

In the linear approximation, we now have

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu} \tag{4}$$

as the definition of our metric tensor. If we define a new field variable

$$\Phi^{\mu\nu} = h^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu} h \tag{5}$$

with the gauge condition

$$\partial_\mu \Phi^{\mu\nu} = 0 \tag{6}$$

then the field becomes

$$-\kappa T^{\mu\nu} = \partial_\lambda \partial^\lambda \Phi^{\mu\nu}. \tag{7}$$

The gauge condition essentially specifies a particular coordinate system for our equations. We will use equation (7) as the gravitational field equation for the remainder of the paper.

Gravity Waves:

We will now consider the special case where $T^{\mu\nu} = 0$, which corresponds to a region free of matter and energy, or free-space. In this case, the field equation becomes

$$\partial_\lambda \partial^\lambda \Phi^{\mu\nu} = 0. \tag{8}$$

We will now look for plane wave solutions to this equation. It can be shown that

$$\Phi^{\mu\nu} = A\varepsilon^{\mu\nu} e^{ik_\alpha x^\alpha} \tag{9}$$

solves this equation, where A is a constant scalar, $\varepsilon^{\mu\nu}$ is a constant tensor, and k_α is a constant vector, so long as we also stipulate that $k^\alpha k_\alpha = 0$, which says that the graviton is massless. We must also specify that $\varepsilon^{\mu\nu} k_\mu = 0$, which means that a gravitational wave is a transverse wave [7]. This solution describes a plane wave with its polarization described by $\varepsilon^{\mu\nu}$ and its direction of propagation and frequency described by k_α. While the solution above is complex, only the real part has any physical interpretation. Since we have a linear differential equation, any linear combination of solutions will also be a solution.

We must now consider the constant polarization tensor $\varepsilon^{\mu\nu}$ and determine the possible forms that this tensor can take. We shall assume that the gravity wave propagates in the $\hat{z}$ direction. Thus we have that

$$k_\alpha = \begin{pmatrix} \omega \\ 0 \\ 0 \\ \omega \end{pmatrix} \tag{10}$$

since this will give us a wave of angular frequency ω propagating in the $\hat{z}$ direction. Starting with this equation, and also considering the energy and momentum carried by a gravitational wave, it can be shown that there are only two possible linearly independent choices for $\varepsilon^{\mu\nu}$ [7]. These are

$$\varepsilon_{\oplus}^{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{11}$$

and

$$\varepsilon_{\otimes}^{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{12}$$

These are the only solutions that carry energy and momentum, though there are other mathematical solutions to the equation. Thus a gravitational wave can have two linear polarizations, and also two circular polarizations,

$$\varepsilon^{\mu\nu} = \varepsilon_{\oplus}^{\mu\nu} - i\varepsilon_{\otimes}^{\mu\nu} \tag{13}$$

and

$$\varepsilon^{\mu\nu} = \varepsilon_{\oplus}^{\mu\nu} + i\varepsilon_{\otimes}^{\mu\nu}, \tag{14}$$

where again, only the real part of the wave has any physical meaning.

We will now consider the effect of a gravity wave on a test mass. It can be shown that there will be no change in momentum for a free particle due to a passing gravity wave [8]. Thus the path that a particle takes, in the coordinate system that we have chosen, is not changed by gravitational radiation. This is a very important result as it tells us that we will not be able to detect a gravitational wave by observing one particle only. We will now consider the effect of a gravity wave of polarization $\varepsilon_{\oplus}^{\mu\nu}$ propagating in the $\hat{z}$ direction on two test masses on the x-axis, separated by a distance of x_0. From above, in the coordinate system that we have chosen, the momentum, and thus the paths, of the particles will not be changed by the passing gravity wave. Let us now consider the separation of the two particles. To do this, we need to use the metric tensor $g_{\mu\nu}$. In Appendix B, I show that if the measured distance between the particles is $x(t)$, then

$$x(t)^2 = \left(1 - \kappa A\cos(\omega t)\right)x_0^2 . \tag{15}$$

Using a Taylor series approximation, which is appropriate since we have small fields, we find that

$$x(t) \approx x_0 - \frac{x_0}{2}\kappa A\cos(\omega t) \tag{16}$$

is the distance between the particles. The gravity wave will have a similar effect on particles along the y-axis, except that here

$$y(t) \approx y_0 + \frac{y_0}{2}\kappa A\cos(\omega t) . \tag{17}$$

Thus the distance between the test masses oscillates as the gravity wave passes. The fact that the momentum of the particles did not change was a trick of the coordinate system we chose. Note that when particles on the x-axis are closer together than x_0, the particles on the y-axis are farther apart than y_0. The effect of a gravity wave on a circle of free particles can be seen in Figure (4).

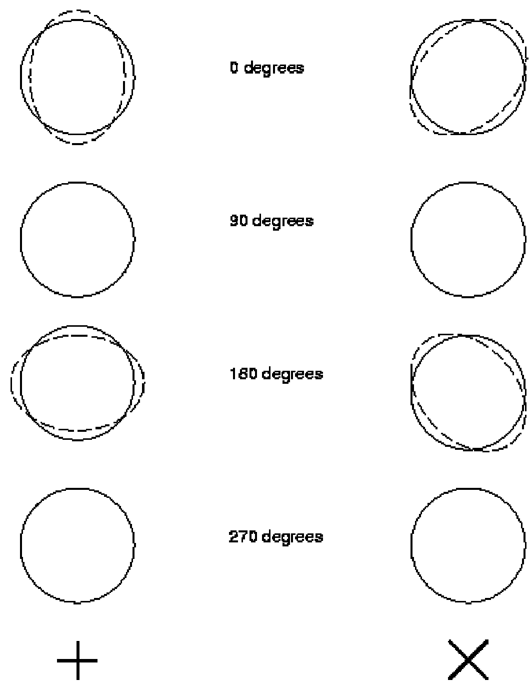

Figure 4: This image shows the effects of a gravity wave on circles of free particles. Note that there are two polarizations of the gravity wave and that they distort the circle of particles in different ways. [http://sepwww.stanford.edu/public/docs/sep75/ray1/Gif/polarities.gif]

Using a more familiar coordinate system we would find that the force between two test masses on the x-axis is

$$F = -\frac{1}{2} m x_0 \kappa A \omega^2 \cos(\omega t), \tag{18}$$

which is the tidal force of the gravity wave [9]. We can also discuss the amplitude of the gravitational wave by looking at change in distance it produces. We say that a gravity wave has strain

$$h = \frac{\Delta x}{x_0} = \frac{\kappa A}{2}, \tag{19}$$

which describes the strength of the wave.

We will now briefly consider the generation of gravitational radiation. A system will produce gravity waves whenever its energy-momentum tensor, $T^{\mu\nu}$, is changing in time. Calculating the energy radiated by gravity waves is very similar to finding the light radiated by a changing charge distribution. In the electromagnetic case, using a first order approximation, the energy radiated depends on how the dipole moment of the charge distribution changes in time. However, there are no mass distributions that will produce a gravitational dipole. Since all masses attract, there is only one charge of mass, which means that a dipole cannot exist. There are however quadrupole mass distributions, and it can be shown that the energy radiated will depend on how the quadrupole moment changes in time. Figure (5) shows a quadrupole distribution of mass that is changing in time, and is thus radiating gravitationally. In electromagnetic theory, the power radiated from a changing quadrupole depends on the square of third time derivative of the quadrupole moment. Similarly, the power of the gravity waves radiated by a changing mass distribution is

$$\frac{dE}{dt} = \frac{4\pi}{45}\left(\frac{\kappa}{8\pi}\right)^2 \dddot{Q}^{kl}\,\dddot{Q}^{kl} \,, \tag{20}$$

where Q^{kl} is the quadrupole moment of the system [10]. It is also possible to determine the maximum strain at some distance R from the source. It turns out that

$$h^{\mu\nu} = \frac{2G}{Rc^4}\ddot{Q}^{\mu\nu} \tag{21}$$

is the largest field that is will be produced by a changing quadrupole [11]. Again, this is similar to electromagnetism, where the maximum magnitude of the electric field depends on the second derivative of the quadrupole moment.

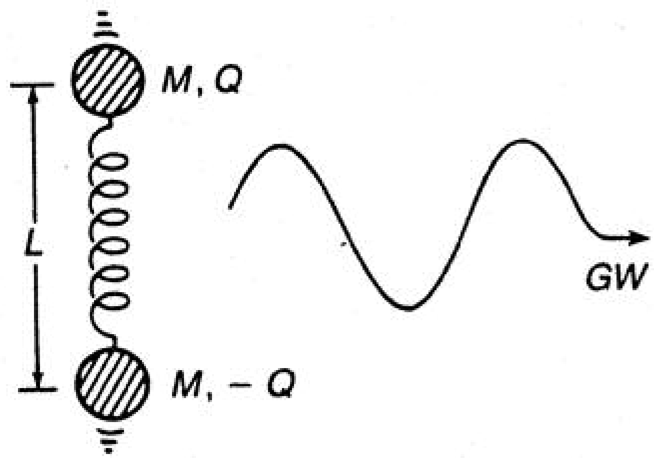

Figure 5: This figure shows a quadrupole distribution of mass that is changing in time, and so it produces a gravity wave, labeled GW. In this case, there are two mass of mass M, separated by distance L vibrating. Alternatively, if the masses where instead charges Q and –Q the system would radiate light. [http://nedwww.ipac.caltech.edu/level5/ESSAYS/Boughn/boughn.html]

Since we know that a changing quadrupole moment of a mass distribution is the source of gravitational radiation, we will now discuss what astrophysical phenomena are likely to produce gravity waves. One theorized source of gravity waves is a neutron star binary system. The mass distribution creates a quadrupole moment and since they are revolving around each other, it changes in both space and time. Thus, we find that the system will radiate gravity waves and lose energy. Indirect observations of this type of radiation have been made using PSR 1913+16 [1]. As the system radiates energy, the stars will fall closer together and will orbit even faster, which causes even more energy to be radiated away. This inspiral towards a collision is likely to produce a chirp of gravity waves as can be seen in Figures (6) and (7), and is one of the events that LIGO hopes to be able to detect. At its target sensitivity, LIGO will be able to see binary neutron inspirals out to about 20 Mpc [12]. The predicted rate of events within 20 Mpc of earth is about one inspiral event every 4 years [13]. There are other theorized sources of gravity waves, such as possible asymmetries in type II supernova, which would likely produce a burst of gravitational waves. Events of this type are expected to occur at a rate of about 1 every 40 years in a Milky Way equivalent galaxy [14]. Theorists also predict that there may be a stochastic background of gravitational radiation stemming from the birth of the universe. A pulsar might also radiate gravity waves if its mass distribution was not symmetric about the axis of its rotation.

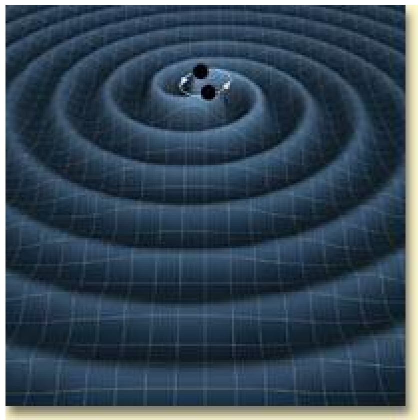

Figure 6: This figure shows the gravity waves that would be produced by a binary neutron star inspiral. [http://spaceplace.jpl.nasa.gov/lisa_fact2.htm]

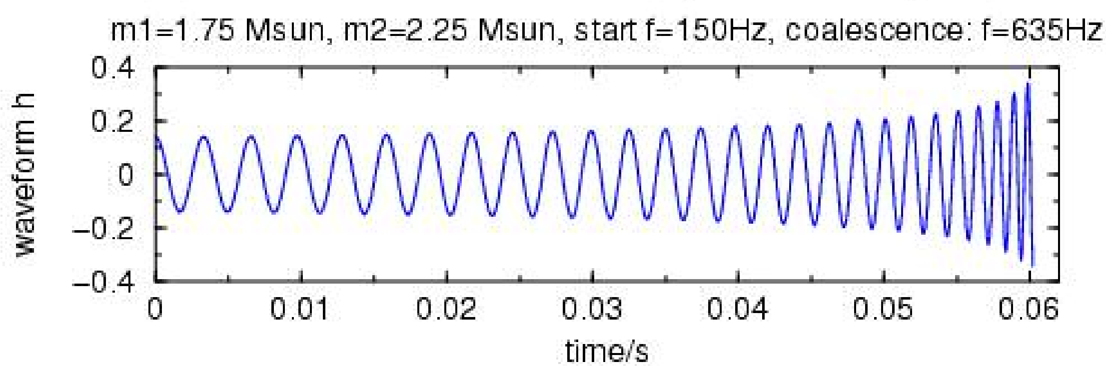

Figure 7: This figure shows the chirp of a compact binary inspiral. Note how the amplitude and frequency of the waves increases as time progresses. [http://gravity.psu.edu/~wolf/PopularScience/Grav_GWs_BHs.html]

The Michelson Interferometer

Michelson Basics:

Since the LIGO detectors are essentially just very large Michelson interferometers with Fabry-Perot cavities in the arms, I shall now discuss the physics of the interferometers. A Michelson interferometer uses the wave properties of light to measure of the changes in distance between points. A basic Michelson interferometer only has 5 parts as can be seen in Fig (8). These are, a laser, a 50-50 beam splitter, two mirrors, and a photo-detector. The laser first hits the beam splitter where it is divided into two beams at right angles to each other. The beams then travel some length L before they reflect off the mirrors and go back to the beam splitter, where they re-combine and go to the photo-detector and back towards the laser.

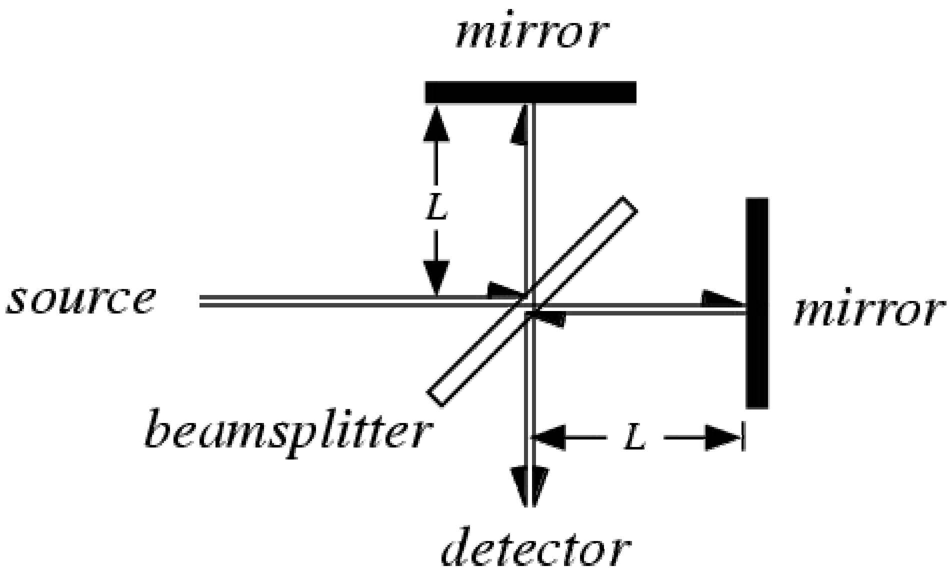

Figure 8: This image is of a basic Michelson Interferometer. The source is for our purposes a laser. In LIGO, L is about 4 km. [http://scienceworld.wolfram.com/physics/]

When the laser light hits the beam splitter it is propagating in the $\hat{x}$ direction and, the equation for the electric field of the light is

$$\vec{E} = E_0 e^{i(k\cdot x-\omega t)}\hat{z}\,. \tag{22}$$

After hitting the beam splitter, though, there will be two beams, one in the $\hat{y}$ direction and one in the $\hat{x}$ direction. The equations will then be

$$\vec{E}_x = \frac{E_0}{\sqrt{2}} e^{i(k_x\cdot x-\omega t)}\hat{z} \tag{23}$$

for the beam continuing in the $\hat{x}$ direction, and

$$\vec{E}_y = -\frac{E_0}{\sqrt{2}} e^{i\left(k_y \cdot y - \omega t\right)} \hat{z} \tag{24}$$

for the beam reflected into the $\hat{y}$ direction. The light is then reflected off the mirrors and comes back to the beam splitter, which works the same way as before, except in the other direction. Thus we find that after hitting the beam splitter again, the electric field of the light propagating in the direction of the photo-detector is

$$E_{\det} = \frac{1}{2} E_0 e^{-i\omega t} \left(e^{i\left(k_x \cdot (2L_x)\right)} - e^{i\left(k_y \cdot (2L_y)\right)} \right). \tag{25}$$

For our purposes, we will assume that $k_x = k_y = \frac{2\pi}{\lambda}$. Using some algebraic tricks, it can be shown that the equation above simplifies to

$$E_{\det} = E_0 e^{i\omega t} \sin\left(\frac{4\pi}{\lambda} \left(L_x - L_y \right) \right). \tag{26}$$

We know that the power being carried by the light is proportional to the electric field squared, so we can say that

$$P_{\det} = P_{laser} \sin^2\left(\frac{4\pi}{\lambda} \left(L_x - L_y \right) \right) \tag{27}$$

is the power of the light at the photo-detector. Thus we see that the power of the light at the photo-detector depends only on the value of $\frac{4\pi}{\lambda}\left(L_x - L_y\right)$, and if its value changes, so will the power of the light that reaches the photo-detector. This value is actually the phase shift of the light between the two arms and in general, I'll represent it as $\Delta\Phi$. Thus for a Michelson interferometer

$$P_{\det} = P_{laser} \sin^2\left(\Delta\Phi\right). \tag{28}$$

Detecting Gravity Waves:

I will now discuss how a Michelson interferometer can be used as a detector of gravitational waves. Note that if $L_x = L_y = L$ we get no light at the anti-symmetric port of the interferometer[3]. Now assume that a gravity wave propagates in the $\hat{z}$ direction with amplitude A, polarization tensor $\varepsilon_{\oplus}^{\mu\nu}$, and frequency ω. From above we found that such a wave will have strain

$$h = \frac{\kappa A}{2} \tag{29}$$

that describes the changes in distances as a result of the wave. I will also assume that the period of the gravity wave is much greater than the time the laser light will be in the arms. With these assumptions we may now proceed to consider how a gravity wave will affect the interferometer. Figure (9) shows an interferometer with a gravity wave of the polarization and propagation direction described above.

[3] The anti-symmetric port of the detector is where the photo-detector is in Figure (8).

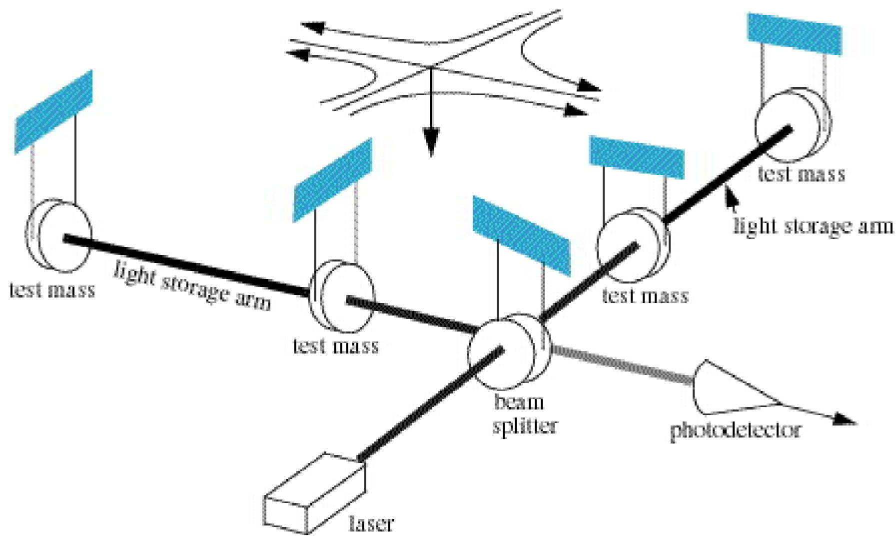

Figure 9: This image shows a gravity wave incident on the interferometer in an ideal orientation. The image also shows the interferometer with the Fabry-Perot cavities. [Barry Barish, *LIGO and the Detection of Gravitational Waves* (LIGO Document LIGO-G000306-00-M, 2000)]

We already saw that the distances between two masses on either the x-axis or the y-axis are

$$x(t) \approx x_0 - \frac{x_0}{2}\kappa A\cos(\omega t) \tag{30}$$

$$y(t) \approx y_0 + \frac{y_0}{2}\kappa A\cos(\omega t). \tag{31}$$

In LIGO, the mirrors are mounted as pendulums so that they behave as free masses and can thus be easily moved by the passing gravity wave. Thus when the gravity wave described above is incident on the interferometer, the lengths of the arms will change so that

$$L_x = L_{x0} - \frac{L_{x0}}{2}\kappa A\cos(\omega t) \tag{32}$$

and

$$L_y = L_{y0} + \frac{L_{y0}}{2}\kappa A\cos(\omega t). \tag{33}$$

Substituting these equations for the lengths of the arms in equation (27), we see that

$$P_{\det} = P_{laser}\sin^2\left(\frac{4\pi}{\lambda}\left(\left(L_{x0} - \frac{L_{x0}}{2}\kappa A\cos(\omega t)\right) - \left(L_{y0} + \frac{L_{y0}}{2}\kappa A\cos(\omega t)\right)\right)\right). \quad (34)$$

Since we have that $L_{x0} = L_{y0} = L$, this simplifies to

$$P_{\det} = P_{laser}\sin^2\left(\frac{4\pi}{\lambda}L\kappa A\cos(\omega t)\right), \quad (35)$$

which means that the power that reaches the photo-detector will oscillate depending on the strain and the frequency of the gravity wave. Thus we can use a Michelson interferometer to detect a gravitational wave, and information about the wave will be encoded in the power of the light detected at the anti-symmetric port.

Another way to do this same problem would have been to consider the phase shift $\Delta\Phi$ that would result from the gravity wave passing through. To find the difference in phase between the two beams of light, simply consider the total difference in time that the beams were in each arm. The light in one arm spends $\frac{2\Delta L}{c}$ longer than normal in the arm, and the light in the other arm spends $\frac{2\Delta L}{c}$ shorter than normal. Thus the total difference in time traveled is $\frac{4\Delta L}{c}$. Since $\Delta\Phi = \frac{2\pi}{\lambda}c\Delta t$ we get that $\Delta\Phi = \frac{8\pi\Delta L}{\lambda}$ and since $h(t) = \frac{\Delta L}{L} = \frac{\kappa A}{2}\cos(\omega t)$, we find that

$$\Delta\Phi = \frac{4\pi}{\lambda}L\kappa A\cos(\omega t). \quad (36)$$

From this we can now say that

$$P_{\det} = P_{laser}\sin^2\left(\frac{4\pi}{\lambda}L\kappa A\cos(\omega t)\right), \quad (37)$$

which is the same answer as the one previously derived.

The LIGO Interferometers:

A simple Michelson interferometer could probably detect a gravity wave if it were strong enough, or if the interferometer were large enough. When considering the noise though, a simple Michelson interferometer, built to a practical scale, would be unable to see gravity waves with realistic strains. The Laser Interferometer Space Antenna [LISA] will be a simple Michelson interferometer with arm lengths of about 5,000,000 km (more on this later). Since a simple interferometer of a size that can be constructed on earth would probably not be able to see gravity waves, the designers of the LIGO interferometers made changes to the simple Michelson design in order to maximize the sensitivity to gravitational radiation. There are several differences between the LIGO interferometers and a simple Michelson interferometer, but the two greatest are the addition of a power recycling mirror between the laser and the beam splitter, and the addition of input mirrors in each arm, creating Fabry-Perot resonant cavities. These input mirrors do not reflect light entering the arms, but they do reflect much of the light that is already in the interferometer arms. A simplified schematic of the LIGO interferometers can be seen in Figure (10).

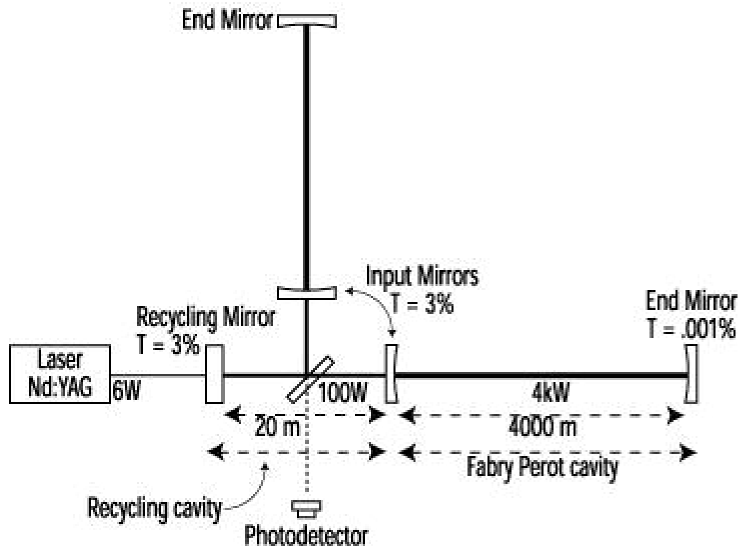

Figure 10: This image shows the interferometer setup of LIGO's 4 km detectors. The two main differences from the simple Michelson interferometer are the addition of a recycling mirror, and the addition of the two input mirrors, which create Fabry-Perot Cavities in the arms of the interferometer. [http://www.spie.org/web/oer/december/dec99/ligo.html]

We will now consider the effects of the Fabry-Perot resonant cavities on the interferometer and, more importantly, their effect on the interferometer's ability to detect a gravitational wave. Since these cavities are resonant structures, the light will create a standing wave in the cavity and the length of each cavity must be very near to $\frac{m\lambda}{2}$, where m is an integer, or the light will destructively interfere with itself in the cavity. The addition of Fabry-Perot cavities in the arms of the interferometers increases the number of trips back and forth the photons in each arm take. We can characterize the cavity by the number of trips that the average photon will take in each arm. In the LIGO interferometers, the average photon takes about 100 trips up and down each arm [12]. Thus the Fabry-Perot cavities have effectively increased the length of the interferometer arms by a factor of 100. With this in mind we will now look at the phase shift of the light in a LIGO interferometer due to a passing gravity wave.

As before, assume a plane wave propagating in the $\hat{z}$ direction with the polarization such that the response of the detector is maximized. I will again assume that the period of the gravity wave is much greater than the time that an average photon spends in the arms. As before, the length of each arm will change by some length ΔL and the time a photon takes to go from the input mirror, to the end mirror, and back to the input mirror will change by $\frac{2\Delta L}{c}$. However, the photon now makes n such trips and so

$$\Delta t = \frac{2n\Delta L}{c} \tag{38}$$

is the total change in the time it spends in the cavity. Thus, from the discussion earlier, this will lead to

$$\Delta\Phi = \frac{4\pi}{\lambda} nL\kappa A\cos(\omega t). \tag{39}$$

This means that the phase shift of the light due to the gravity wave is increased by a factor of n, greatly increasing the sensitivity of the interferometer. At the same time, the length of the arms needed to attain a certain sensitivity is reduced.

The other change evident in Figure (10) is the addition of a power recycling mirror. This mirror is similar to the input mirrors in that it allows nearly all the light to enter the interferometer, but it reflects most of the light exiting the interferometer. By using a power recycling mirror, the laser power in the interferometer can be vastly increased. Changing the power of the light in the interferometer has a large effect on the noise; the power recycling mirror reduces the shot noise though it does lead to greater radiation pressure noise.

Sources of Noise in LIGO

Noise is one of the plagues facing the LIGO experiment, and much of the current research is devoted to lowering the noise in the interferometers. One of the goals of the LIGO interferometers is to be able to detect a gravity wave with strain of about 10^{-22} at 150 Hz [12]. This means that even though the arms of the interferometer are 4 km long, the mirrors will only move about 10^{-19} meters. LIGO must then be able to know the positions of the mirrors to within this order of magnitude, and be able to detect perturbations of this size. This means that the noise in the interferometer must be kept at extremely low levels. Figure (11) shows the predicted noise curves for an advanced LIGO interferometer and from it one can see the contributions from various different noise sources.

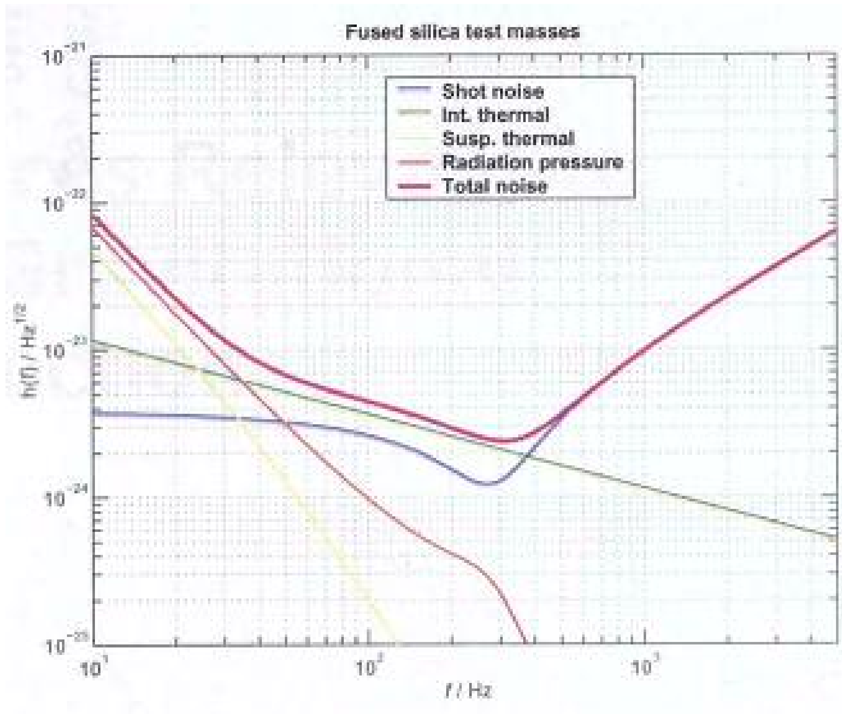

Figure 11: This figure shows the predicted noise curves for an advanced LIGO interferometer. The Noise is similar in the first generation LIGO interferometers, only greater by about an order of magnitude. LIGO will be most sensitive from about 50 Hz to about 1000 Hz. [Shanti Rao, *Thermal Noise Sources Relevant to Interferometric Gravitational Wave Detection* (LIGO Document LIGO-G010016-00-R, 2001)]

Shot Noise:

Shot noise is one of the major sources of noise in the LIGO interferometers, and is in fact the limiting noise source for higher frequency ranges [15]. Shot noise comes from the inherent uncertainty in the phase of the light, and since LIGO is in effect measuring the phase shift of the light, an uncertainty in the phase will produce uncertainty in the phase shift. In Appendix C, I show that if we have a laser beam with an average of N photons arriving per second, then the uncertainty in the number of photons that will arrive in any given second is $\sqrt{N}$. We will now use Heisenberg's uncertainty principle to examine the uncertainty that results in the phase shift. First note that $\Delta\Phi = \omega\Delta t$ and that $\Delta E = \hbar\omega\Delta N$ which we can then insert into Heisenberg's uncertainty relationship, which says that $\Delta E\Delta t \geq \frac{\hbar}{2}$ to get that

$$\Delta\Phi\Delta N \geq \frac{1}{2}. \tag{40}$$

We can also use the energy of the photon to say that $\Delta N = \sqrt{\frac{nI}{\hbar\omega}}$, where I is the intensity of the laser beam when it hits the beam splitter. Since we already have an equation that relates $\Delta\Phi$ to the change in position of one mirror, Δx, we can now say

$$\frac{4\pi}{\lambda}\Delta x\sqrt{\frac{nI}{\hbar\omega}} \geq \frac{1}{2}. \tag{41}$$

Solving for Δx we get

$$\Delta x \geq \frac{c}{4}\sqrt{\frac{\hbar}{nI\omega}}. \tag{42}$$

Because the uncertainty in the position of one mirror is uncorrelated to the uncertainty in the position of another mirror, the uncertainties add in quadrature and so $\Delta h = \frac{2\Delta x}{L}$, which means that

$$\Delta h \geq \frac{c}{2L}\sqrt{\frac{\hbar}{nI\omega}}. \tag{43}$$

This result is very important as it says that the uncertainty in the strain decreases as the intensity of the light increases. The reason for this may be described in the following way. Each photon makes a measurement of the mirrors' positions, and the more measurements taken, i.e. the more photons bounced off the mirrors, the lower the uncertainty in the actual measurement. Thus a greater intensity of light hitting the mirrors will lead to a lower uncertainty in the phase of the light, and lower uncertainty in the strain. Another important aspect of shot noise is the fact that it is independent of the frequency at which measurements are taken. Thus for some laser intensity, the shot noise sets a lower limit on how much the noise can be decreased. Figure (12) shows the optical readout noise, which is the noise from both radiation pressure noise and shot noise.

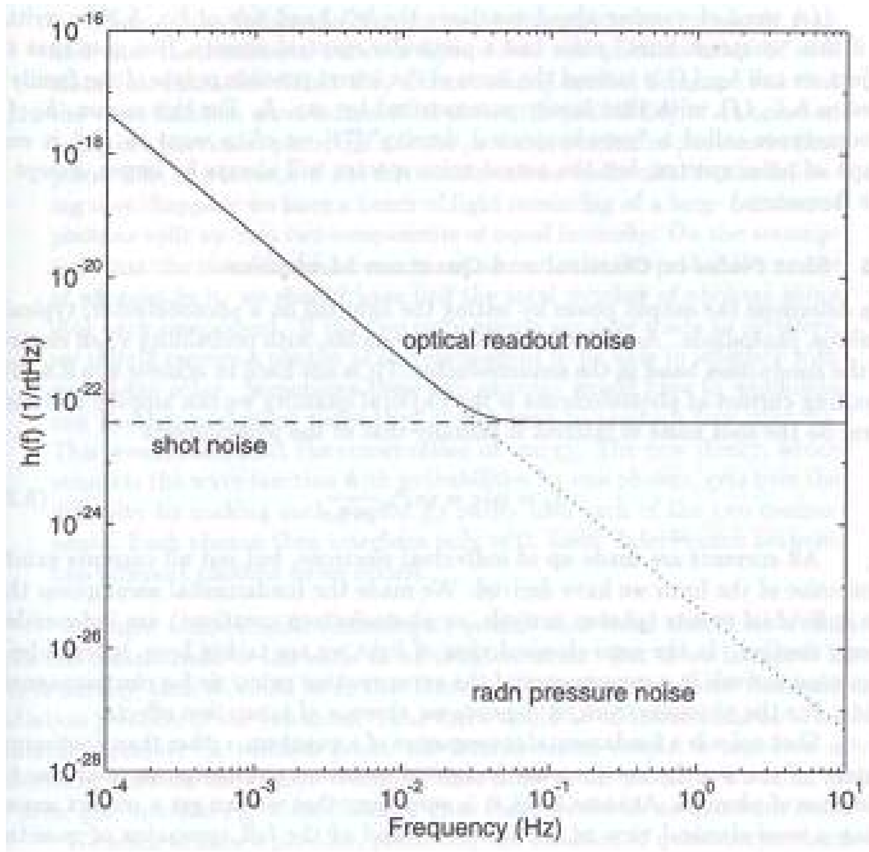

Figure 12: This is plot shows the noise in the strain as a function of frequency. The calculation of this noise was done for a simple Michelson interferometer with 500 km arms, 5 watts of laser power, and 10 kg mirrors. [Peter R. Saulson, *Fundamentals of Interferometric Gravitational Wave Detectors* (World Scientific Publishing Company, Singapore, 1994) p. 79]

Radiation Pressure Noise:

One of the many important aspects of quantum mechanics that comes into play in the interferometers is the fact that photons have momentum, and may impart a force on an object. Thus when they are reflected by a mirror, the mirror feels some force due to the reflected photons. Since there is an uncertainty in the number of photons striking the mirror for some period of time, there is also an uncertainty in the force on the mirror. We have that the force F imparted by a beam of light reflecting normally off of a surface is

$$F = \frac{I}{c} \tag{44}$$

and so the uncertainty in this force for one of the LIGO mirrors is

$$\Delta F = \frac{\Delta I}{c}. \tag{45}$$

It can be shown that $\Delta I = \sqrt{nI\hbar\omega}$. The intensity at each mirror, however, is half of the intensity at the beam splitter so thus we get that

$$\Delta F = \frac{1}{c}\sqrt{\frac{nI}{2}\hbar\omega}. \tag{46}$$

This leads to some uncertainty in the position of the mirror, Δx. Using the properties of a harmonic oscillator, it can be shown that $\Delta x \geq \frac{\Delta F}{(2\pi f)^2 m}$, where m is the mass of the mirror and f is the frequency at which the mirror is oscillating[4]. From this we get that

$$\Delta x \geq \frac{1}{c(2\pi f)^2 m}\sqrt{\frac{nI}{2}\hbar\omega}. \tag{47}$$

Using this equation to find the uncertainty in the strain, we get

$$\Delta h \geq \frac{1}{2Lc\pi^2 f^2 m}\sqrt{\frac{nI}{2}\hbar\omega}. \tag{48}$$

As before, it is important to note that increasing the intensity of the light has an effect on the noise; in this case, increasing the intensity actually increases the noise. Another important point about radiation pressure noise is the fact that it depends on the frequency

[4] Remember that the mirror is suspended as a pendulum and is thus a harmonic oscillator.

at which one is measuring. This effect can be seen in the plot of radiation pressure noise in Figure (12).

The Standard Quantum Limit:

The fact that radiation pressure noise increases with the intensity of the light and shot noise decreases with the intensity of the light leads to a fundamental limit on how accurately one can measure the position of the mirror with photons. Since the radiation pressure noise and the shot noise are uncorrelated[5], they add in quadrature. Thus we find that $\Delta h = \sqrt{\Delta h_{rad}^2 + \Delta h_{shot}^2}$, and so the minimum Δh is when $\Delta h_{rad} = \Delta h_{shot}$. Now we can simply set up this equation and solve for I so that these two uncertainties will be equal. Doing this we find that noise will be a minimum when

$$I = \sqrt{2}\frac{c^2 f^2 m\pi^2}{n\omega}. \tag{49}$$

Calculating the total optical read-out noise using this laser intensity gives

$$\Delta h = \frac{1}{2^{3/4}\pi L f}\sqrt{\frac{\hbar}{m}} \approx \frac{1}{\pi L f}\sqrt{\frac{\hbar}{m}}. \tag{50}$$

This is a very important result because it indicates that this is the limit of measurement due to uncorrelated radiation pressure noise and shot noise, and it does not depend on anything other than the frequency at which the measurement is made, the length of the interferometer, and the mass of the mirrors. LIGO's mirrors are currently 10 kg masses, though new mirrors made of sapphire may have a mass of about 40 kg.

Strange as it may seem, there are ways around the standard quantum limit. The reason for this lies in the equation

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[5] These two sources of noise are uncorrelated because shot noise comes from the uncertainty in the phase of the light and radiation pressure noise comes from the uncertainty in the intensity of the light.

$$\Delta h = \sqrt{\Delta h_{rad}^2 + \Delta h_{shot}^2} \quad , \tag{51}$$

which is not an accurate statement if the radiation pressure noise and the shot noise are correlated. One way of correlating the noise from radiation pressure with the shot noise is to use what are called squeezed states of light. We have already seen that $\Delta\Phi\Delta N \geq \frac{1}{2}$. What happens with squeezed states is that one can reduce $\Delta\Phi$ while increasing ΔN, or vice versa, and in doing so, create a correlation between the shot noise and the radiation pressure noise. To create this correlation, one would inject squeezed light into the anti-symmetric port of the interferometer. By doing this, it is possible to achieve measurements more accurate better than the standard quantum limit [16]. In the case where radiation pressure noise and shot noise are correlated, the total noise will be

$$\Delta h = \sqrt{\Delta h_{rad}^2 + \Delta h_{shot}^2 + 2\,\mathrm{cov}(\Delta N, \Delta\Phi)}\,, \tag{52}$$

where $\mathrm{cov}(\Delta N, \Delta\Phi)$ is a measure of the correlation between ΔN and $\Delta\Phi$. Another way around the standard quantum limit is by using quantum non-demolition techniques. Instead of measuring the position of the mirrors, one could measure their momentum. Since the momentum operator commutes with itself at different times, it is possible to observe the momentum of the mirrors without influencing that momentum [16]. For this reason, measuring the momentum of the mirrors is a way in which a future interferometer could beat the standard quantum limit.

Thermal Noise:

The thermal noise for the LIGO interferometers is one of the larger noise sources at low frequencies. Thermal noise in general is perhaps best explained by looking at the source of the noise, which is the random chaotic motion of molecules. The equi-partition theorem says that every normal mode of oscillation will be excited with energy $\frac{k_B T}{2}$ due to thermal energy. However, while this tells us the total energy in each mode, it does not specify how the noise will be distributed over different frequencies.

What I will now present is know as the fluctuation-dissipation theorem, and since it is a rather complex idea, I will only touch on the basics of the theorem. The idea is that one can describe the noise in a system based on the rate of loss of energy from the system. We start with the basic idea that we may write the equation of motion of a harmonic oscillator as

$$F = Z(f)v, \tag{53}$$

where F is the external force on the oscillator, v is the velocity, and Z is the impedance [17]. The impedance of an oscillator is generally complex and is defined by the equation above. All of these are considered as functions of the frequency at which the oscillator moves. It can then be show that

$$x_{th}^2(f) = \frac{kT}{\pi^2 f^2} \mathrm{Re}(Y(f)), \tag{54}$$

where $Y(f) = Z^{-1}(f)$ and is called the admittance [17]. What this tells us is that if we can determine the losses in the system, due to the impedance, we can also find the thermal noise of that system. In LIGO there are many sources of this loss, including friction in the mounting of the pendulum and stretching of the wires supporting the mirrors. However, using the above formalism, we can accurately characterize the thermal noise.

Another way in which to look at the thermal noise is by using the quality factor Q of the oscillator. The quality factor measures the size of dissipation around the resonant frequency of the oscillator and is defined by

$$Q \equiv \frac{f_0}{\Delta f}, \tag{55}$$

where f_0 is the resonant frequency and Δf is the full width at half the maximum of the power of the noise. A high quality factor means that most of the noise will be at the resonant frequency, which in turn says that thermal noise from that mode of oscillation will not contribute much noise at other frequencies. A pendulum works out to be a remarkably high quality oscillator because the principle restoring force, gravity, is lossless. While the internal friction in the wires suspending the mirrors will cause some noise, it is much less than if the wires were responsible for the restoring force [18]. Figure (13) shows the strain noise due to the pendulum suspension of mirrors in an interferometer. There are several other sources of thermal noise, such as the internal modes of the mirrors themselves, but this noise can be analyzed using the techniques described above.

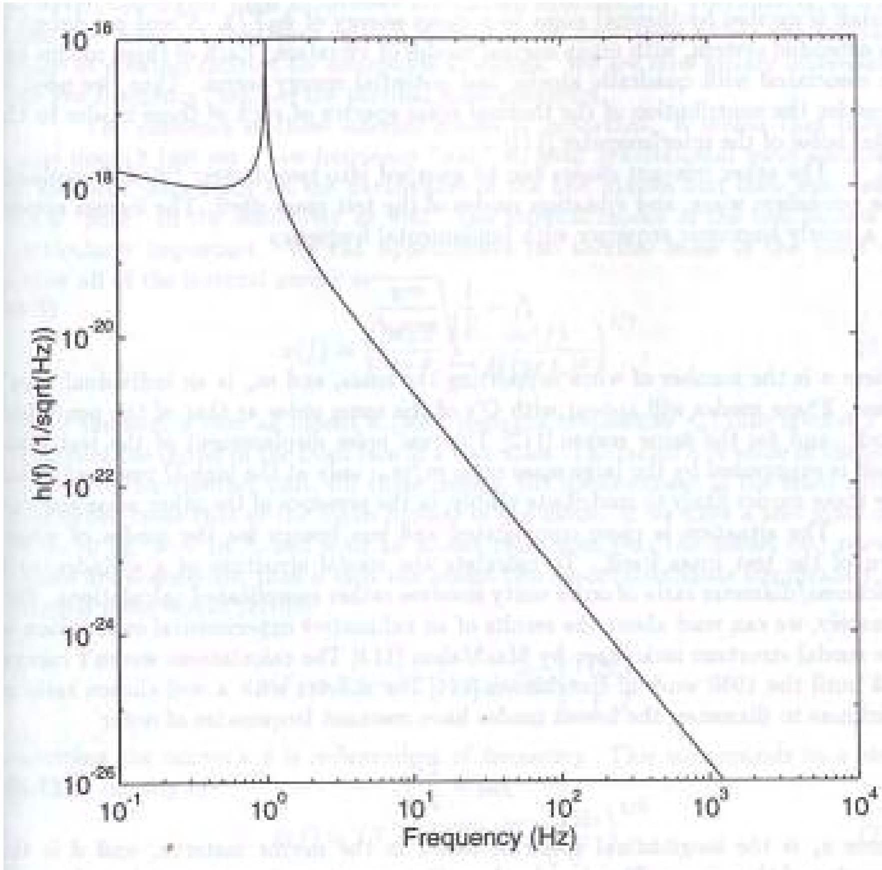

Figure 13: This figure shows the noise in an interferometer due to the thermal noise of the pendulum suspension of the mirrors. The pendulum has resonant frequency 1 Hz, mass 1 kg, and is suspended by four tungsten wires of diameter .12 mm. Here the interferometer has L=4 km. [Peter R. Saulson, *Fundamentals of Interferometric Gravitational Wave Detectors* (World Scientific Publishing Company, Singapore, 1994) p. 123]

Future Work and Conclusion

For all the physics taking place inside the detectors, each one is a very low noise Michelson interferometer. In fact, much of the research currently being done has to do with lowering the noise in the detectors even further. The work with squeezed light is one example of current research that is aimed at reducing the noise in LIGO. Currently, LIGO is sensitive to gravity waves with strains of about 10^{-21} which means that the experiment is very close to reaching the design sensitivity [19]. The sensitivities of all the LIGO interferometers, along with the target sensitivities can be seen in Figure (14).

Once the detectors reach their target sensitivities, it is expected that they will be upgraded to allow for strain sensitivities better than 10^{-23}. This would allow the detectors to see binary neutron star inspirals out to a distance of about 350 Mpc. The predicted rate of binary inspiral events out to this distance is about 4 events per day [13].

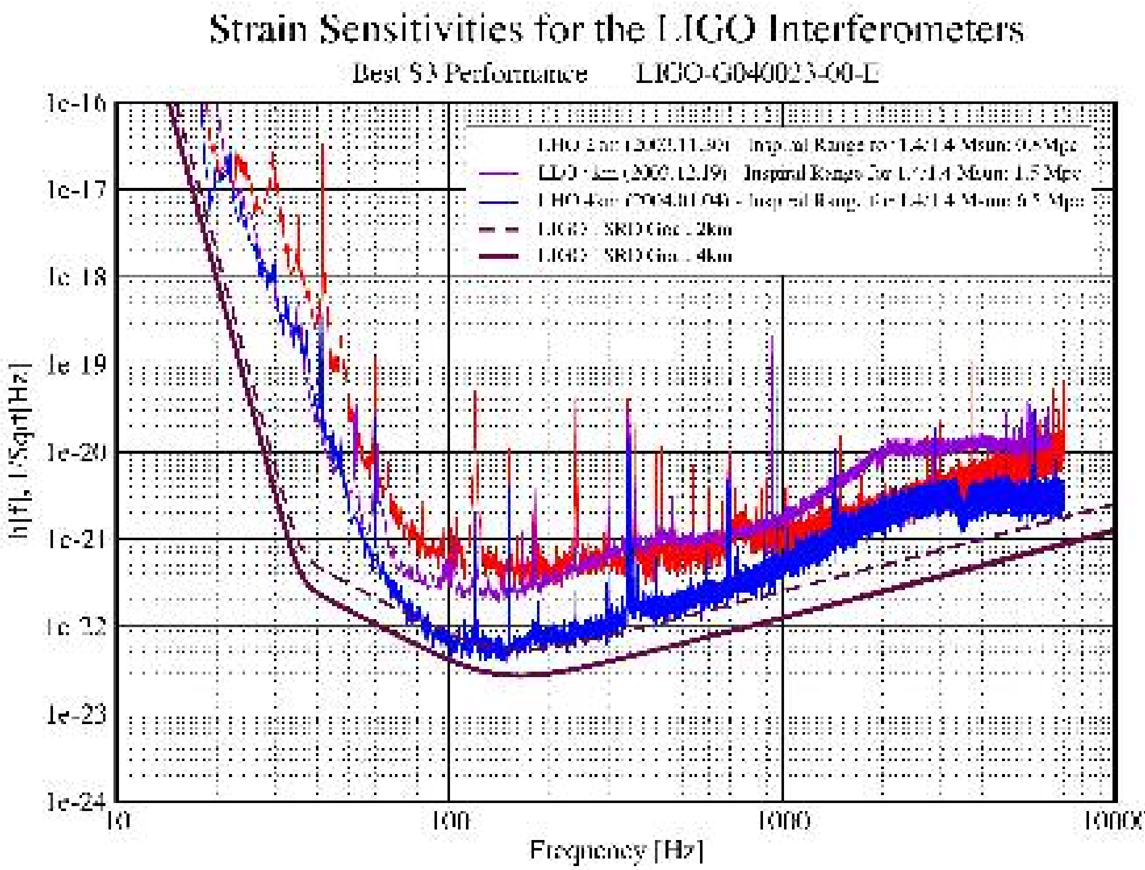

Figure 14: This image shows the best strain sensitivities of the three LIGO interferometers during the S3 run. The run was from 31 October 2003 through 9 January 2004. [LIGO-G040023-00-E, 2004]

Another area that has quite a lot of active research is data analysis. This deals with analyzing the signal that the detector outputs, and determining if a gravity wave is present, and if so, what caused it. There are four data analysis groups, Inspiral, Burst, Pulsar, and Stochastic Background, since these are the predicted principle sources of gravitational radiation. There is also research being done by theorists to predict the waveform of a gravity wave from a particular source. Another important area of research is LISA, which will search for gravity waves at frequencies lower than LIGO's sensitivity curve allows. Figure (15) shows the different frequency ranges in which LIGO and LISA will operate. Because LIGO and LISA are sensitive in different frequency ranges, they will be searching for types of events, the same way that optical astronomers and x-ray astronomers look for different events.

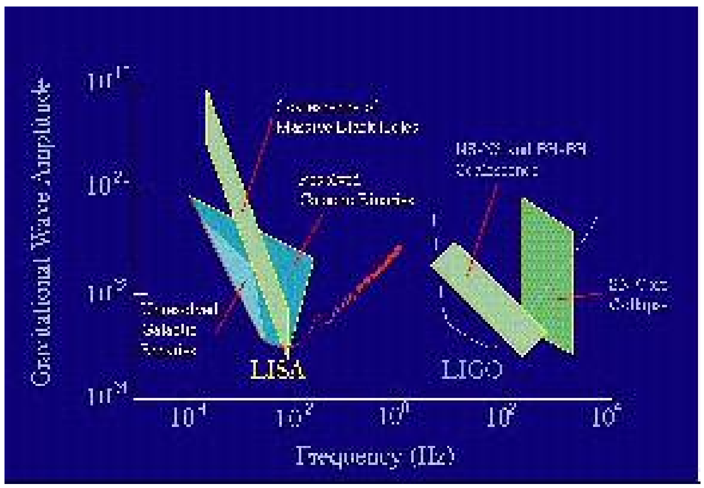

Figure 15: This figure shows the frequencies at which LIGO and LISA will operate. The left axis is actually the strain, while the curves show the noise in each device. The planes show frequency and amplitude of expected signals. [http://www.srl.caltech.edu/lisa/graphics/05.LIGO.LISA.jp]

The importance of gravity wave detection is not just to serve as a proof of Einstein's theory of general relativity. Gravity wave astronomy is likely to allow people to observe events we could not otherwise see. For example, with a system of four or more detectors separated by a couple thousand kilometers, physicists could triangulate the location of a source of gravity waves. When paired with optical astronomy, this might allow physicists to study events using both light and gravity waves, providing a more complete observation. As can be seen in Figure (16), the LIGO detectors have been situated at quite some distance from one another for just this reason. Also, the LIGO interferometers function in concert with a growing group of others around the world, which may further provide the ability to localize sources of gravitational radiation. The first detection of gravity waves will certainly be a milestone, but physicists also hope to use gravity waves to study the objects or events that produced them. It may become possible to observe what goes on in the core of a type II super-nova using gravity waves. Neutron star binary inspiral events might also serve as a standard candle, which would aid in determining the Hubble constant [20]. Finally, information from a stochastic gravity wave background could give clues as to the very first moments of the universe. For all of these reasons, gravity wave astronomy is likely to become a very important field.

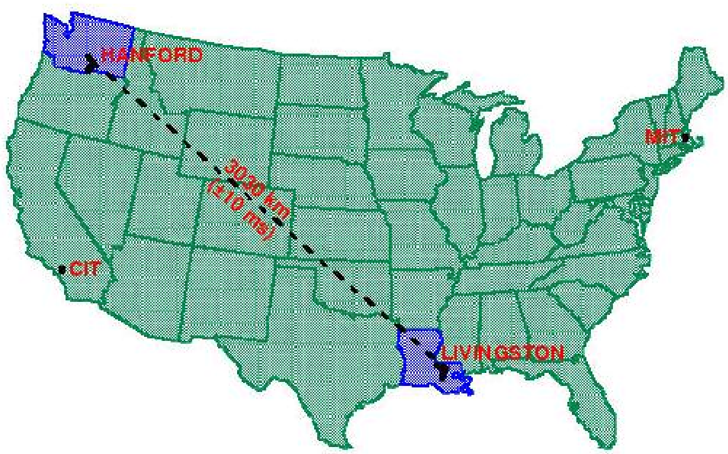

Figure 16: This image shows the locations of the LIGO interferometers, and the distance between them. The detectors were deliberately place a large distance from one another to aid in localizing the source of gravitational waves detected. [Barry Barish, *LIGO and the Detection of Gravitational Waves* (LIGO Document LIGO-G000306-00-M, 2000)]

Conclusion:

The search for gravitational radiation is currently an extremely exciting area of physics research, and while there is good evidence for the existence of gravity waves they have not been directly observed. The LIGO experiment has the ambitious goals of detecting gravitational radiation and using incoming signals to examine the universe through gravity wave astronomy. While the detectors themselves are very complex devices, at their hearts they are simply Michelson interferometers. Similarly, though the noise curve for LIGO is complicated, many of the individual noise sources are explainable using relatively simple physics. Gravity waves, when detected, are likely to open up a new way of looking at the universe, and future observations may reveal objects and phenomena that have never even been considered before.

Appendix A: Notation

This section discusses the notation used in this paper. First consider a vector in spacetime. The components will be represented using superscripts so $x^0 = t$ and $x^1 = x$ and so on. Here the symbol A^μ will represent a vector in spacetime. This paper also uses the Einstein summation convention. Whenever a repeated index appears, this translates as a sum over all time and space. In this paper

$$B_\alpha A^\alpha = \sum_{\alpha=0}^{3} B_\alpha A^\alpha \ . \tag{56}$$

In taking partial derivatives, the following notation will apply,

$$\frac{\partial f}{\partial x^\alpha} = \partial_\alpha f \ . \tag{57}$$

Finally, a dot such as $\dot{f}$ will indicate a derivative with respect to proper-time.

Appendix B: Derivation of Changes in Distance Caused by a Gravitational Wave

The distance between two points when a gravity wave passes is of great importance. For this derivation, I'll be looking at a gravity wave with

$$\Phi^{\mu\nu} = A\varepsilon^{\mu\nu} e^{ik_\alpha x^\alpha} \ , \tag{58}$$

where $\varepsilon^{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$, and $k_\alpha = \begin{pmatrix} \omega \\ 0 \\ 0 \\ \omega \end{pmatrix}$. Thus for this case, we have

$$\Phi^{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & Ae^{ik_\alpha x^\alpha} & 0 & 0 \\ 0 & 0 & -Ae^{ik_\alpha x^\alpha} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{59}$$

Since we have $\Phi^{\mu\nu} = h^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu} h$, we can also say that

$$h^{\mu\nu} = \Phi^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu}\Phi, \tag{60}$$

where Φ is the trace of $\Phi^{\mu\nu}$. In this case however, $\Phi = 0$, and so $h^{\mu\nu} = \Phi^{\mu\nu}$. We know that $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$ which means that for this gravity wave we have

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 + \kappa A e^{ik_\alpha x^\alpha} & 0 & 0 \\ 0 & 0 & -1 - \kappa A e^{ik_\alpha x^\alpha} & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{61}$$

Now I'll examine two objects on the x-axis in the $z = 0$ plane. Assuming they had an initial separation of x_0 the metric tensor tells us that their separation will be

$$x(t)^2 = -g_{11} x_0^2 = (1 - \kappa A e^{i\omega t}) x_0^2 = (1 - \kappa A \cos(\omega t)) x_0^2. \tag{62}$$

Appendix C: Statistics

Photon arrival times obey Poisson statistics, which means that if on average μ photons arrive per unit length of time, then the probability of r photons arriving in that length of time is described by the probability distribution

$$P(r) = \frac{\mu^r e^{-\mu}}{r!}. \tag{63}$$

I'll use the standard deviation σ as the uncertainty in a particular measurement. Using the equation

$$\sigma^2 = \langle r^2 \rangle - \langle r \rangle^2 \tag{64}$$

where

$$\langle f(r) \rangle = \sum_{r=0}^{\infty} f(r) P(r) \tag{65}$$

is the expectation value for $f(r)$, we find that for the Poisson distribution,

$$\sigma^2 = \mu . \tag{66}$$

Thus for an average of μ photons arriving each second, we will have an uncertainty of

$\sigma = \sqrt{\mu}$.

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18) Ibid. p. 121-124
19) *Strain Sensitivities for the LIGO Interferometers: Best S3 Performance* (LIGO Document LIGO-G040023-00-E, 2004)
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Annotated Bibliography

Hans C. Ohanian, Remo Ruffini, *Gravitation and Spacetime*, 2nd ed. (W. W. Norton and Company, New York, New York, 1994).

This book was my main source for the general relativity in this paper. I found it to be quite readable and to give excellent derivations. This book is most useful if one takes the time to understand the tensor notations and has some knowledge of differential geometry. This book also contains an excellent chapter on special relativity that I found to be quite useful. The book also contains a useful section on gravity wave detectors.

Richard S. Millman, George D. Parker, *Elements of Differential Geometry* (Prentice-Hall, Upper Saddle River, New Jersey, 1997).

I used this book to look at the differential geometry in general relativity, and to try to get a better grasp of some of the ideas presented in the above book. This book is quite dense and assumes a pretty solid background in mathematics. It is a good book for someone who has a through mathematical background and wants to understand the mathematical underpinnings of general relativity.

Eugene Hecht, *Optics*, 4th ed. (Addison Wesley, San Francisco, California, 2002).

This is another excellent book that I used mostly as reference for the section on Michelson interferometers. I also found it to be quite useful in discussing the Fabry-Perot cavities. This book is somewhat short on derivations.

Peter R. Saulson, *Fundamentals of Interferometric Gravitational Wave Detectors* (World Scientific Publishing Company, Singapore, 1994).

This book was the principle source for much of this paper. A good, if basic, description of gravitational radiation is at the beginning, and then the book goes into the physics of an interferometric detector. The book also contains excellent sections on several different sources of noise. I found chapters 1-7 to be extremely useful, and I read selectively from the others.

David G. Blair, *The detection of gravitational waves* (Cambridge University Press, Cambridge, England, 1991).

This is a book that has a lot of information on gravity wave detectors of all kinds. The later sections were on interferometric detectors. This book is denser and harder to read than Saulson, and it does not present nearly as much introductory material. It does, however, have a lot of information not presented in Saulson. I used it mostly to gain a deeper understanding of material presented elsewhere.

James K, Blackburn, *The Laser Interferometer Gravitational Wave Observatory Project* (LIGO Document LIGO-P960031-C-E, 1997).

This paper was a good source for information about LIGO specifically. The paper presented a broad overview of much of the physics in LIGO including a good section on noise. There are a lot of equations given to the reader that are not derived so I generally did not spend much time on the mathematics in this paper. The first few sections were excellent and greatly helped me in my understanding of the project as a whole.

Thomas Corbitt, Nergis Mavalvala, *Quantum Noise in Gravitational-wave Interferometers* (LIGO Document LIGO-P030067-00 R, 2003).

I used this paper as a source for my noise section. There is a brief discussion of radiation pressure noise and shot noise, and the Standard Quantum Limit is discussed in some detail. The majority of the paper deals with ways of beating the Standard Quantum Limit.

Gary H. Sanders, David Beckett, *LIGO: An Antenna Tuned to the Songs to Gravity* (Sky and Telescope, 41-48, October 2000).

This was good introductory piece. I used it to get started and to get some ideas as to the kinds of things I wanted to include in my paper. This would be a good article for someone with a minimal background in physics who was interested in learning a bit about LIGO.

William R. Leo, *Techniques for Nuclear and Particle Physics Experiments,*

2nd ed. (Springer-Verlag, New York, New York, 1994)

I used this book as a source in discussing the statistics of photon arrival times and error propagation. The book is easy to read and is mostly devoted to the function of nuclear and particle detectors. I found chapter 4, *Statistics and the Treatment of Experimental Data*, to be extremely useful.