Goodsell Gazette

Carleton College
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The newsletter for the Carleton mathematics and statistics community

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So You Want to be a Math or Stat Major

Are you a first or second year student contemplating a math or math/stat major? If so, join us for our annual prospective majors meeting at 4:00pm on Tuesday, February 14 in CMC 206. Department chair Deanna Haunsperger will provide information about the major and several faculty members will talk about cool math/stat topics. There will also be appropriately themed snacks. Hope to see you there!

Konhauser Memorial Problemfest

On Saturday, February 25, Macalester College will host the 20th annual Konhauser Memorial Problemfest, named after the late Macalester professor and legendary problem poser Joe Konhauser. In this contest teams of up to three students get three hours (9 am to noon) to work together on a set of ten challenging and intriguing math problems. Afterwards participants have lunch together while the solutions are graded, and the results are announced. The winning team gets to take the "pizza trophy" home to their college for the year. A Carleton team won last year, so you can see the pizza trophy next to the chairs outside Sue's office. Needless to say, it would be great to keep the pizza trophy here at Carleton for another year.

If you would like some practice with past Konhauser problems, drop by the problem solving group, which meets on Wednesdays, 4:30-6 pm, in CMC 328. To sign up for this year's Konhauser, contact Mark or Eric as soon as possible; three people can sign up as a team, but individuals are also welcome to express interest, and we might be able to help you find teammates.

Summer Research UC Berkeley

The UC Berkeley Geometry, Topology, and Operator Algebras RTG Summer Research Program for Undergraduates 2012 is a unique 8 week program for up to 12 undergraduates to explore cutting edge mathematics and get involved with active research problems. Participating students will complete an intensive four week course on either contact and symplectic geometry or homological algebra and then investigate open-ended problems in these fields. In their free time, students will have the opportunity to explore Berkeley, San Francisco, and the wonderful Bay Area.

Applications are due March 12, 2012.
For additional information, visit:
www.math.berkeley.edu/~gardiner/berkeleymathsummer.html

Summer Research University of Minnesota

Seeking motivated undergraduate students for an eight week summer program at the University of Minnesota, Minneapolis, in the dynamics of pattern formation. This program is ideal for students contemplating graduate school or a career in mathematics that would like to gain experience performing mathematical research.

No previous research experience is expected. The only firm requirement is an undergraduate course in differential equations, although higher level coursework and a familiarity with or interest in learning Matlab would be helpful. Participating students must be US citizens or permanent residents.

Applications will be considered until the positions are filled. Offers begin as of March 1. For additional information, visit: www.math.umn.edu/~scheel/

Spring Term Course Offerings

Math 241: Ordinary Differential Equations
Prerequisite: Math 232 or consent of instructor
Professor: Mark Krusemeyer
Class Time: 3a

Description: In calculus you may well study separable first-order differential equations for a bit, but that's just the tip of the iceberg! In any field where mathematics is applied, you are likely to find equations relating unknown functions and their derivatives. Over the centuries, following the lead of Newton, Leibniz, and the Bernoullis, mathematicians have come to grips with many such equations. Naturally, they prefer to get exact solutions if possible, and we'll look at some of the systematic methods (and a few of the clever *ad hoc* tricks) that have been developed to find solutions. On the other hand, there are times when finding an exact solution is too difficult, or even potentially misleading - for instance, because the mathematical model that leads to the differential equation is imprecise to begin with. In such cases, it is often best to concentrate on the qualitative behavior of solutions; for example, you might try to predict what will happen in the long run.

In this course, you'll find plenty of calculus-style computation, including ample opportunity to brush up on your techniques of integration (*Mathematica* can help with some of that), but also a few theoretical discussions, some geometric ideas, and a bit of mathematical modeling. The textbook we'll be using, which was written by a close (younger!) relative, does not presuppose much linear algebra, but concepts from linear algebra, ranging from vector spaces of functions through linear transformations and kernels to eigenvalues and eigenvectors, will be mentioned and used with some regularity in class.

Math 245: Applied Regression Analysis
Prerequisite: Math 215 or 275
Professor: Katie St. Clair
Class Time: 3a

Description: Model building is a fundamental idea in statistics. In an intro stats class you've learned some basic techniques for modeling a response as a linear function of one explanatory variable (simple linear regression). In this second stats course you will learn more advanced techniques for building regression models that can include many explanatory variables (multiple regression) or a categorical response (logistic regression). We will apply these techniques to explore how air pollutants might affect mortality, whether sex plays a role in determining a worker's salary, and how a national tragedy was predicted by a regression model. This course emphasizes model building and checking techniques and statistical writing. We will meet in the stats lab and use the free statistical software R. As the title suggests, this is an applied course so you will be working with new data sets each week, and you can expect to be a seasoned R user by the end of the term!

Math 295: Seminar in Set Theory
Prerequisite: Math 236 or consent of instructor
Professor: Gail Nelson
Class Time: 2a

Description: Is there a set of all sets? If you allow the collection of all sets to be considered a set, the result, as you encountered in Math 236, is Russell's paradox. So what exactly can be considered a set? We attempt to answer this by working with a set (pun intended!) of axioms. We'll begin by building familiar number systems (the natural numbers, the rationals and the reals) from these axioms. Then our ambitions get really grandiose, as we proceed to construct successively larger infinite numbers, and finally, the entire mathematical "universe." Along the way, we will encounter cardinal numbers, ordinal numbers, and transfinite induction.

Since its inception, set theory has been a source of beautiful and, at times, disturbing results. Attempts to reduce all of mathematics to the language of sets have provided insight into unifying underlying structure. But set theory has also led to strange and counterintuitive "alternate realities" for mathematics.

Math 315: Statistical Computing
Prerequisite: Math 275
Professor: Laura Chihara
Class Time: 2a

Description: Many problems in statistics such as evaluating complex integrals (for Bayesian inference) or estimating parameters of a sampling distribution can be tackled using simulation. In this course, we will study statistical techniques that can be handled computationally. We will cover methods for generating random variables, Monte Carlo integration and variance reduction including importance sampling, and Markov chains Monte Carlo methods. Depending on time, we may also study the bootstrap and jackknife, density estimation, etc. We will use R extensively.

Math 341: Fourier Series & Boundary Value Problems
Prerequisite: Math 241
Professor: Gail Nelson
Class Time: 4a

Description: Partial differential equations are equations involving partial derivatives (naturally!). They are also the principal mathematical tools used to explore such diverse disciplines as acoustics, aerodynamics, fluid dynamics, geophysics, heat transfer, meteorology, and quantum mechanics. In this course, we will use the heat, wave and Laplace equations as our guides as we explore questions such as: When and how can we solve these equations? Very few of these equations can be solved explicitly. Even if we can't find explicit solutions of a PDE, can we learn anything about the properties of its solutions? In particular, how well can we approximate solutions with or without computers?

As the course title suggests, we will be exploring what is known as the Fourier method for solving such problems. This method not only leads to Fourier series, but also to a theory of orthogonal functions and their associated generalized Fourier series.

Math 351: Functions of a Complex Variable
Prerequisite: Math 211
Professor: Mark Krusemeyer
Class Time: 5a

Description: What happens to calculus when you replace the real variable x by the complex variable $z = x + iy$ and the real-valued function $y = f(x)$ by the complex-valued function $w = f(z)$? Wonderful things! The statement "f is differentiable" becomes more powerful, while the idea of integration becomes more flexible - you can now integrate along various paths in the complex plane. Not only does this lead to much beautiful mathematics, but, and this might surprise you, a lot of this material can be applied to "real" (pun intended) mathematical and physical problems in which nary an imaginary number occurs. For instance, we should see how to compute some useful improper integrals of functions, such as $\cos(x^2)$, that don't have an antiderivative in closed form, and if time permits we'll see how to apply complex transformations to solve problems of heat conduction in the plane. On the other hand, we'll see a proof of the "Fundamental Theorem of Algebra", which states that any nonconstant polynomial with complex coefficients has a (complex) root - and thus any such polynomial factors completely into linear factors.

This course has connections with many other upper-level math courses, and veterans of such courses should enjoy those connections. Thus there is an argument for taking Math 351 late in your Carleton career. However, if you're not into delayed gratification, Math 211 is really the only prerequisite for the course.

Despite what the academic catalog and historical precedent would suggest, Math 351 WILL BE OFFERED AGAIN NEXT YEAR.

Math 352: Abstract Algebra II
Prerequisite: Math 342
Professor: Eric Egge
Class Time: 4a

Description: During the first half of the term we will focus on group theory, studying group actions, the Sylow theorems, the finite simple groups and (if things go quickly) a way of "factoring" groups which is analogous to factoring integers. Much of this work will be motivated by a simple question: can we classify all groups?

We will not have enough time to actually settle this question, even though we will see some of the tools one might want to use. During the second half of the term we will study Galois theory, which concerns a deep connection between groups and fields. Galois theory is a powerful tool for studying polynomials. In fact, by the end of the term we should know enough to show that it's impossible to trisect all angles using just a straightedge and compass.

We should also be able to explain to a fifth grader why there is no "quadratic formula" for fifth degree polynomial equations. No actual fifth graders will be harmed in this process.

The Tour Continues

Next week, the Tour of Mathematics (Friday, February 17, 3:30 PM, CMC 206) will feature Helen Wong speaking about "DNA Knots." (Katie St. Clair's talk this week is on "Sampling in Population Networks.")

A Sneak Peek at Next Year's Schedule

In a few weeks we will be finalizing the schedule of mathematics and statistics courses which will be offered next year. At this point, this is still a tentative schedule, but if it would make it easier to plan this spring's classes by what's offered above Structures next year, be our guest to take a peek:

Fall
Probability (2 sections)
Real I
Combinatorics
295 or 395 TBD
Differential Geometry

Winter
ODEs
Applied Regression
Statistical Inference
Real II
Abstract I
Chaos

Spring
ODEs
Applied Regression
Prob/Stat Seminar
Abstract II
Methods of Teaching Math
Complex
Number Theory
Fourier Series

PROBLEMS OF THE WEEK

1. Let a and b be any real numbers. Define a sequence (x_n) by
$x_0 = a$, $x_1 = b$, $x_2 = f(x_0, x_1)$,
$x_3 = f(x_1, x_2)$, $\cdots$, $x_{n+2} = f(x_n, x_{n+1})$, $\cdots$,

where f is the function given by

$$f(x, y) = \ln\left(\frac{e^x + e^y}{2}\right).$$

a) Show that unless $a = b$, no two numbers in the sequence are equal.

b) Show that the sequence has a limit, and find this limit (in terms of a and b).

2. Consider the parabola $y = x^2$. For different points P in the plane, there may be different numbers of normal lines to the parabola that pass through P.

a) Show that there is always at least one normal line, and that there are at most three normal lines, to $y = x^2$ that pass through any given point P.

b) Show that there are exactly two normal lines to $y = x^2$ that pass through $Q = (4, \frac{7}{2})$, and find and sketch the set of all points Q with this property.

c) Are there any such points Q for which *both* coordinates of Q are integers?

Last week's first problem was solved both by Dylan Peifer and by Justin Troyka, and the B.B.O.P. (in CMC 217) awaits a visit from Justin. So far there are no on-campus solutions for the second problem, but John Snyder in Oconoowoc solved both problems. Good work, all! Meanwhile, my own solutions for the problems from the first three weeks of this term have been posted in the hallway outside CMC 217.

- Mark Krusemeyer

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