

# Goodσελλ Gazette

Carleton College  
Northfield MN 55057

The newsletter for the Carleton mathematics and statistics community

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## *Equitability and the GINI Index*

Next Tuesday, September 27<sup>th</sup>, Benedict Visiting Professor of Mathematics, Frank Farris, will be speaking on the mathematical components of the GINI Index, a number economists use to express the degree to which resources are equitably distributed in a nation. The main mathematical tool of the index is a simple one: the definite integral that gives the area between two curves. Please join us for the talk in Olin 141 at 7pm.

## *Bain & Company Recruiting for Management Consulting Positions*

Bain & Company is a management consulting firm based in Boston, but with offices around the country. This year, they are making a major push to hire liberal arts graduates for their Associate Consultant (AC) positions. They are looking for candidates with strong problem-solving skills, a results oriented track record, strong leadership experience and a passion for consulting. The deadline to apply, **October 1<sup>st</sup>**, is fast approaching. Applicants are expected to have strong academic records, excellent analytical skills and a high level of motivation and commitment. A cover letter, resume and transcript will be necessary to complete the application. To access the application and to obtain more information about the company, go to [www.joinbain.com](http://www.joinbain.com) and select the Associate Consultant position. Additionally, feel free to contact Alec Roman '10 at [alec.roman@bain.com](mailto:alec.roman@bain.com) with any questions.

## *The Northfield Undergraduate Mathematics Symposium Returns!*

Long awaited, highly anticipated, often imitated but never duplicated, the Northfield Undergraduate Mathematics Symposium returns Thursday afternoon at St. Olaf. Students from both Carleton and St. Olaf will give talks on their summer research, and we'll have a break along the way for pizza and great conversation. The Symposium is from 3:30 to 8 pm on Thursday, September 29 at St. Olaf College. From 3:30 to 5:30 talks will be in Regents Hall 310, followed by dinner and conversation in Regents Hall 410. Talks after dinner will also be held in room 410. Take a look at the attached sheet for an idea of the topics you'll learn about at the symposium.

## *Annual All Science and Math Poster Session*

Mark your calendars and join us for the annual celebration of student scholarship in science and math. The Annual All Science and Math Poster Session is on Wednesday, October 26, 2011, in room 236 of the Weitz Center from 3:45 p.m. to 5:15 p.m.

We welcome all students who have done research, at Carleton or elsewhere, to present and we encourage all to attend. This event has grown to over 60 posters presented and last year over 100 people attended. For questions, please contact Ellen Haberoth. Sponsored by: The Carleton Chapter of Sigma Xi, the Howard Hughes Medical Institute (HHMI), and the Carleton Interdisciplinary Science & Math Initiative (CISMI).

### *Biostatistics Open House*

The Biostatistics Division at the University of Minnesota is hosting an open house on Friday, October 7, 2011 for students interested in their graduate program. The half day session will feature information about the MS, MPH and PhD programs, research opportunities, job prospects, financial aid and application process. Visit their web site for more information: <http://www.sph.umn.edu/biostatistics>

An RSVP is required. Also, if you would like a ride, contact Laura Chihara (lchihara).

### *Seven Rivers Undergraduate Research Symposium*

Viterbo University's 9<sup>th</sup> annual Seven Rivers Undergraduate Research Symposium is slated for Friday, November 11, 2011, from 11 a.m. to 5 p.m. This symposium showcases the research and creative works of undergraduate students from a variety of disciplines. All interested students and faculty on your campus are encouraged to attend. This year's keynote speaker will be Dr. Margaret Lowman, Director of the Nature Research Center of the North Carolina Museum of Natural Sciences, and North Carolina State University Research Professor. Meg Lowman pioneered the science of canopy ecology. For 30 years, she has designed hot-air balloons and walkways for treetop exploration to solve mysteries in the world's forests, with special expertise on the links between insect pests and ecosystem health. There is no fee for symposium registration or presentation.

The registration deadline is Friday, October 28, 2011 (note: all attendees must register) and the abstract submission deadline is Wednesday, November 2, 2011. Online registration and abstract submission forms as well as additional information about the event can be found online at: <http://www.viterbo.edu/sevenrivers/>.

### *PROBLEMS OF THE WEEK*

1. Let  $E$  be an ellipse in the plane. Describe the set  $S$  of all points  $P$  outside the ellipse such that the two tangent lines to the ellipse that pass through  $P$  make a right angle (at  $P$ ). Show why your description is correct. (As an example, if  $E$  is a circle, it's not hard to see that  $S$  is also a circle, with radius  $\sqrt{2}$  times the radius of  $E$ .)
2. Suppose you pick the one million entries of a  $1000 \times 1000$  matrix independently and at random from the set of all ten digits (that is, each entry is one of  $0, 1, 2, \dots, 9$ ). Is the determinant of the matrix you get more likely to be even or odd, and why?

So far, no solutions to last week's problems have come in. Looking forward to getting some of those and/or solutions to the new problems above! (Reminder: To be acknowledged in Friday's *Gazette*, solutions should reach my box in the CMC by Tuesday evening.)

- Mark Krusemeyer

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Problems of the Week:	Mark Krusemeyer
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## *Mathematics Symposium Abstracts:*

### **Prediction by Iterative Supervised Principal Components** **Sen Zhao (Carleton)**

In this statistical genetics project, we tried to classify people based on their gene expressions. We found that existing methods, including LASSO, Tibshirani's Near-est Shrunken Centroids, Supervised Principal Components (PCA), and 2-Means did not work well with datasets which were unbalanced and had high mislabeling rate. However, unbalancedness and mislabeling are common in the real world. Therefore, we developed an iterative method which extends the algorithm of Supervised PCA proposed by Bair, Hastie, Paul, and Tibshirani (2006). This method worked better than traditional Supervised PCA in terms of error rate, sensitivity, and specificity.

### **A Method of Word Recognition** **Reid Whitaker (Carleton)**

Automated word recognition is extremely critical in a wide variety of human machine interactions. Some examples include automated voice answering systems, automated dialing, and direct voice input in airplanes. A method of word recognition was developed using Fourier and wavelet analysis to determine the error of an unknown word compared to the known words in a small library. Semi-reliable results were achieved at identifying an unknown word from a speaker not included in the library.

### **The Isoperimetric Inequalities on Constant Gauss Curvature Surfaces** **Xin Chen (Carleton)**

We give a new proof for the isoperimetric inequalities on spheres and hyperbolic planes using metacalibration. Unlike the classical optimization approach calculus of variations, metacalibration compares competitors directly to the proposed minimizer via vector fields and the divergence theorem. It paves the way to solve open problems such as multiple bubbles and isoperimetric problems with boundary on constant Gauss curvature surfaces.

### **Restricted Symmetric Signed Permutations** **Andy Hardt (Carleton)**

The symmetry group  $D_4 + \mathbf{Z}_2$  acts on the set of signed permutations by rotations, reflections, and bar operations (flip the sign of each entry). Following Egge's work on unsigned permutations, we enumerate the signed permutations that, given a symmetry subgroup  $H$  and a set  $R$  of length-2 signed patterns, are invariant under  $H$  and avoid  $R$ . Mansour and West began this work by enumerating the signed permutations that avoid  $R$ , not taking symmetries into account. Duker and Mansour continued by enumerating signed involutions that avoid  $R$ . This talk considers the remaining subgroups of  $D_4 + \mathbf{Z}_2$ , thus completing the enumeration. The resulting sequences include the Catalan numbers and the central binomial coefficients, and some of them are given recursively. We present some of these results in the talk, and the rest are in our accompanying paper.

### **Classifying $f$ -vectors of Manifolds with Boundary** **Jonathan Hahn (Carleton)**

A 3-dimensional manifold with boundary, such as a ball or solid torus, can be represented by sets called simplicial complexes made of faces of various dimension—points, line segments, triangles, and tetrahedrons. For a given simplicial complex, we can count the number of faces in each dimension, and encode this information in its  $f$ -vector. So far, we know all the  $f$ -vectors for very few 3-dimensional manifolds with boundary. In this talk, I will discuss how to characterize  $f$ -vectors for the simplest case, a ball.

### **Extremal Graphs Without 4-Cycles or, Why It's Hip To Have No Squares** **Frank Firke (Carleton)**

Extremal graph theory deals with two main questions: what is the maximum number of edges a graph on  $n$  vertices can have before it must contain a given subgraph, and what graphs achieve that maximal condition? In this talk we will examine the question of extremal graphs when we forbid 4-cycles. While the problem remains unsolved in general, we will consider a result that answers the question for infinitely many  $n$ . The problem, while graph theoretic in nature, also has a significant connection to finite geometry that will be touched on briefly.

**Triangle Puzzles and Quantum Cohomology**  
**Erik Wyatt (St. Olaf)**

Combinatorial representations of geometric objects can be used to find their intersections. We will show how to compute a variation on Littlewood-Richardson coefficients that describes all lines passing through two varieties.

**The Dirichlet Process Prior in a Hierarchical Catch-Effort Model for Animal Abundance**  
**Prasit Dhakal and**  
**Jun Young Park (Carleton)**

The Dirichlet Process Prior (DPP) in Bayesian Statistics offers useful insight in studying animal abundance if heterogeneity of animal abundance has been unobserved. Consider  $N_i$ , the animal abundance in region  $i$ , follows a Poisson distribution with mean of  $A_i \varphi_i$ , where  $A_i$  is the area of region  $i$ . Then it would be one way to assume  $\varphi_i$  to come from one single distribution in the model, such as  $N(\mu_\varphi, \sigma_\varphi)$ , but the model with the DPP would not make this assumption and estimate the animal abundance better. This talk includes a brief review of previous work (without the DPP) and its comparison with the model with the DPP.

**$\tau$ -Factorization in the Integers**  
**Nathan Bishop (St. Olaf)**

This talk will serve as an introduction to the Theory of  $\tau$ -Factorization, a generalization of Factorization Theory. The theory hinges on the relation  $\tau$ : let  $D$  be an integral domain,  $U(D)$  be the units of  $D$ , and  $D^\#$  be defined as  $D \setminus \{0\} \cup U(D)$ . Then  $\tau$  is any relation defined on  $D^\# \times D^\#$ . For the purposes of this talk, we will consider  $\tau$  to be the relationship of modular equivalence. After laying the foundation of the theory, we will demonstrate several results, including properties of the integers under a  $\tau$ -relation.

**Enumerating Partitions of Generalized Stars**  
**Robert Crandall (St. Olaf)**

In graph theory, a topic of study is graph partitions of a graph  $G$ : graphs which can be obtained from  $G$  by removing vertices and/or edges. We wish to enumerate the number of partitions of certain classes of graphs. Interesting sequences arise in this study, particularly since the partitions of a path  $P_n$  on  $n$  vertices correspond to the integer partitions of  $n$ . Counting these partitions is an NP-hard problem, particularly since many partitions of  $G$  will be isomorphic. We will examine two types of generalized stars, graphs obtained by identify-

ing the end vertices of any number of paths of specific lengths, and will present and demonstrate formulas for the number of partitions of these graphs.

**Non-negativity of Generalized Immanants of Monomial Positive Matrices**  
**Vladimir Sotirov (St. Olaf)**

On one hand, a monomial positive matrix is a (square) matrix whose entries are polynomials in some number of indeterminates, satisfying the property that the determinant of every square submatrix is a polynomial in those indeterminates with non-negative coefficients (perversely, we say that the determinant is monomial positive function on the square submatrices). Monomial positive matrices arise naturally as the weight matrices of planar networks with indeterminate weights. On the other hand, generalized immanants are functions on matrices which generalize the determinant by replacing the sign function (in the expansion of the determinant as sum) with an arbitrary function on the symmetric group. Tautologically, the determinant of a monomial positive matrix is monomial positive; little is known, however, about monomial positive immanants of monomial positive matrices. By brute force computation in Maple, it is known that for up to  $n = 5$  the cone of monomial positive immanants of  $n \times n$  monomial positive matrices is finitely generated (unexpectedly, the number of generators for  $n = 5$  is 121, rather than  $120 = 5!$ ). Since brute force becomes computationally infeasible for  $n = 6$ , I will describe in this talk a more systematic approach, which reduces the computational complexity to the point where a human could not only verify in an hour or two that there are finitely many generators for  $n = 6$ , but also determine in a day or two of careful computation by hand (or a few seconds of execution on a computer after a week of careful programming) exactly how many generators there actually are.

**Thompson's group  $F$ : A New Generating Set**  
**Amelia Stonesifer (St. Olaf)**

Thompson's group  $F$  was introduced by Richard Thompson in the 1960's in connection with questions in logic. It has since found applications in many areas of mathematics including algebra, logic and topology, and its metric properties with respect to the standard generating sets,  $X_n$ , have been studied heavily. In this talk, we introduce a new family of generating sets, which we denote as  $Z_n$ , use "wave diagrams" as tools to establish a length formula for the word metric with respect to  $Z_1$  and apply the word length formula to demonstrate that  $F$  is not almost convex and  $F$  has a dead end of depth at least 1 with respect to  $Z_1$ .