Special Sets of Vertices in Paley Graphs

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Overview

1. Paley Graphs
   - Constructing $P(q)$
   - Partitioning $P(q)$

2. Tight Sets

3. Affine Planes
   - $P(q)$ in $AG(2, q)$
   - Results

4. Current Work

5. Summary and Acknowledgements
Introduction

**Definition**

A **graph** is a collection of vertices and edges, where each edge is composed of exactly two vertices.
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Definition

A graph $G$ is strongly regular if for parameters $(v, k, \lambda, \mu)$: there exist $v$ vertices each adjacent to $k$ others, such that any 2 adjacent vertices share $\lambda$ common neighbors and any 2 nonadjacent vertices share $\mu$ common neighbors.
Definition

The **adjacency matrix** of a graph is a 0-1 matrix indexed by the graph’s vertices that keeps track of which vertices are adjacent in the graph.

The adjacency matrix for a strongly regular graph will have exactly 3 eigenvalues $k, \theta_1, \theta_2$ that give us information about the structure of the graphs.
Vaguely, a **finite field**, $\mathbb{F}_q$, is a set of $q$ elements in which addition, multiplication, subtraction, and division are defined and have properties similar to the real numbers.

- All fields have order $p^k$ where $p$ is prime. Fields of the same order are isomorphic.
- $\mathbb{Z}_{13}$, the integers 1 to 13, are a field.
- $\mathbb{Z}_9$, the integers 1 to 9, do not form a field, although $\mathbb{F}_9$ does exist.
The Paley Graph

Definition

A **Paley graph** \( P(q) \) is a graph

- with vertex set \( \mathbb{F}_q \) where \( q = p^n \equiv 1 \) (mod 4) for prime \( p \)
- vertices \( u,v \) are adjacent iff \( (u - v) \) is a nonzero perfect square in \( \mathbb{F}_q \)
The Paley Graph

**Definition**

A **Paley graph** \( P(q) \) is a graph

- with vertex set \( \mathbb{F}_q \) where \( q = p^n \equiv 1 \pmod{4} \) for prime \( p \)
- vertices \( u,v \) are adjacent iff \( (u - v) \) is a nonzero perfect square in \( \mathbb{F}_q \)

Paley graphs are **strongly regular** with parameters:

\[
(v, k, \lambda, \mu) = \left( q, \frac{q - 1}{2}, \frac{q - 5}{4}, \frac{q - 1}{4} \right)
\]

and eigenvalues:

\[
(k, \theta_1, \theta_2) = \left( \frac{q - 1}{2}, \frac{-1 + \sqrt{q}}{2}, \frac{-1 - \sqrt{q}}{2} \right)
\]
Example: P(13)

Figure 1: P(13) is a strongly regular graph with parameters (13,6,2,3).
Example: $P(81)$

Figure 2: $P(81)$ has a total of 1620 edges
Cliquies and Independent Sets

**Definition**

A **clique** is a subset of a graph’s vertices such that every pair of vertices in the subset are adjacent.
Clique and Independent Sets

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An **independent set** is a subset of a graph’s vertices such that no two vertices within the subset are adjacent.
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*Paley graphs can be partitioned by their cliques or independent sets.*
Figure 3: The clique decomposition made up of 6 cliques of order 3 in P(9), where x is a generator for the field.
Independent Sets Example

Figure 4: $P(9)$ with two independent sets of order 3 highlighted
The eigenvalues of a graph, \( k > \theta_1 \geq 0 > \theta_2 \), give bounds on the order of cliques and independent sets. If \( T \) is a set of vertices such that each vertex is adjacent to \( \alpha \) others, we have

\[
\theta_2 + \frac{(k - \theta_2)|T|}{v} \leq \alpha \leq \theta_1 + \frac{(k - \theta_1)|T|}{v}
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If \( \alpha = 0 \) then \( T \) is an independent set and if \( \alpha = |T| - 1 \) then \( T \) is a clique.
The eigenvalues of a graph, $k > \theta_1 \geq 0 > \theta_2$, give bounds on the order of cliques and independent sets. If $T$ is a set of vertices such that each vertex is adjacent to $\alpha$ others, we have

$$\theta_2 + \frac{(k - \theta_2)|T|}{\nu} \leq \alpha \leq \theta_1 + \frac{(k - \theta_1)|T|}{\nu}$$

If $\alpha = 0$ then $T$ is an independent set and if $\alpha = |T| - 1$ then $T$ is a clique.

Not all cliques and independent sets meet these bounds. Those which do meet the bounds have special properties.
Motivating Tight Sets

Given a "special" clique in a strongly regular graph, any vertex outside the clique will be adjacent to $\alpha'$ vertices inside the clique, where $\alpha'$ is a constant.
Motivating Tight Sets

Given a "special" clique in a strongly regular graph, any vertex outside the clique will be adjacent to $\alpha'$ vertices inside the clique, where $\alpha'$ is a constant.

*Can these "special" cliques and independent sets be generalized?*
Figure 5: Cliques of order 3 in P(9) are "special", with $\alpha' = 1$. 
Motivating Tight Sets

Definition

Given a vertex set $V$ and a subset $T \subseteq V$, where on average each vertex in the set is adjacent to $\alpha$ others in the set and each vertex not in the set is adjacent to $\alpha'$ vertices in the set, we call $\alpha$ the interior intersection number and $\alpha'$ the exterior intersection number.
Motivating Tight Sets

**Definition**

Given a vertex set $V$ and a subset $T \subseteq V$, where on average each vertex in the set is adjacent to $\alpha$ others in the set and each vertex not in the set is adjacent to $\alpha'$ vertices in the set, we call $\alpha$ the **interior intersection number** and $\alpha'$ the **exterior intersection number**.

**Example:** In a clique with 3 vertices, $\alpha = 2$ because each vertex is connected to 2 others.
Introduction to Tight Sets

The adjacency matrix of a strongly regular graph (SRG) has 3 distinct eigenvalues where $k > \theta_1 \geq 0 > \theta_2$. Given a subset $T$ of the vertices with intersection number $\alpha$, we obtain the following:

$$\theta_2 + \frac{(k - \theta_2)|T|}{V} \leq \alpha \leq \theta_1 + \frac{(k - \theta_1)|T|}{V}$$
Introduction to Tight Sets

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For the Paley Graph $P(q^2)$,

$$\frac{1}{2}(q - 1)(\frac{|T|}{q} + 1) \leq \alpha \leq \frac{1}{2}(q - 1)(\frac{|T|}{q} + 1)$$
Introduction to Tight Sets

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$$\frac{1}{2}(q - 1)(\frac{|T|}{q} + 1) \leq \alpha \leq \frac{1}{2}(q - 1)(\frac{|T|}{q} + 1)$$

If the upper or lower bound on $\alpha$ is achieved, we have a ”tight interlacing” and $T$ is a tight set.

The subgraph induced by a tight set is always $\alpha$-regular (for SRGs).
Because of their eigenvalues, only Paley graphs of order $q^2$ where $q \in \mathbb{Z}$ contain tight sets, so we will refer to $P(q^2)$.

**Definition**
A set $T$ of vertices in $P(q^2)$ is a **tight set Type I** if each vertex of $T$ is adjacent to exactly $\alpha = \frac{1}{2}(q + 1)(\frac{|T|}{q} - 1)$ other elements of $T$. **Type I** generalizes tight independent sets and **Type II** generalizes tight cliques.
Tight Sets

Because of their eigenvalues, only Paley graphs of order \( q^2 \) where \( q \in \mathbb{Z} \) contain tight sets, so we will refer to \( P(q^2) \).

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A set \( T \) of vertices in \( P(q^2) \) is a **tight set Type I** if each vertex of \( T \) is adjacent to exactly
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\alpha = \frac{1}{2}(q + 1)(\frac{|T|}{q} - 1)
\]
other elements of \( T \).

**Definition**

A set \( T \) of vertices in \( P(q^2) \) is a **tight set Type II** if each vertex of \( T \) is adjacent to exactly
\[
\alpha = \frac{1}{2}(q - 1)(\frac{|T|}{q} + 1)
\]
other elements of \( T \).

Type I generalizes tight independent sets and Type II generalizes tight cliques.
Parameters of Tight Sets

**Theorem**

If $T$ is a tight set in $P(q^2)$, $|T| = cq$ for some $c ∈ ℤ$ where $1 ≤ c ≤ q$.

**Definition**

We refer to a tight set of order $cq$ as a tight set of parameter $c$.

**Example:** In $P(q^2) = P(25)$, a tight set of order 5 is of parameter 1, whereas a tight set of order $2 \times 5 = 10$ is of parameter 2.
Tight Sets: Example

Figure 6: A tight set of Type II of parameter 1 in P(25). Note: This is a tight set and a clique, so it is a tight clique.
Finding Tight Sets in Paley Graphs

- Method 1:
  - Search through all possible subgraphs $G$ of size $cq$ where $c \in \mathbb{Z}$ and test if:
  $$\forall s \in G, \alpha = |N(s) \cap T| = \theta + \frac{(k-\theta)|T|}{v}$$
Finding Tight Sets in Paley Graphs

- **Method 1:**
  - Search through all possible subgraphs $G$ of size $cq$ where $c \in \mathbb{Z}$ and test if:
  $$\forall s \in G, \alpha = |N(s) \cap T| = \theta + \frac{(k-\theta)|T|}{v}$$

- **Method 2:**
  - Search for characteristic vectors in the eigenspace of the graph’s adjacency matrix.
Finding Tight Sets in Paley Graphs

**Definition**

A **characteristic vector** of a graph is a 0-1 vector that corresponds to a subset of vertices.

**Remark**: For a graph with \( v \) vertices, characteristic vectors live in the vector space \( \mathbb{R}^v \).

For a strongly regular graph, \( \mathbb{R}^v = E_k \bigoplus E_{\theta_1} \bigoplus E_{\theta_2} \)
Finding Tight Sets in Paley Graphs

**Definition**

A **characteristic vector** of a graph is a 0-1 vector that corresponds to a subset of vertices.

**Remark**: For a graph with $v$ vertices, characteristic vectors live in the vector space $\mathbb{R}^v$.

For a strongly regular graph, $\mathbb{R}^v = E_k \oplus E_{\theta_1} \oplus E_{\theta_2}$

We can find characteristic vectors for:

- a tight set of Type I in the basis for $E_k \oplus E_{\theta_2}$
- a tight set of Type II in the basis for $E_k \oplus E_{\theta_1}$
### Tight Sets in Paley Graphs

Recall: For a tight set $T$ of parameter $c$, $|T| = cq$. 

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<th>P(25) Type II</th>
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## Tight Sets in Paley Graphs

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<td><strong>Total</strong></td>
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Recall: For a tight set $T$ of parameter $c$, $|T| = cq$. 
Preliminary Results

Theorem

\( P(q^2) \) has the same number of Type I and Type II tight sets of each parameter.
Preliminary Results

**Theorem**

\[ P(q^2) \text{ has the same number of Type I and Type II tight sets of each parameter.} \]

**Theorem**

In \( P(q^2) \) there are the same number of tight sets (type I or II) of parameter \( m \) (\( m \neq q \)) as there are of parameter \( q - m \).
### Tight Sets in Paley Graphs Revisited

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Recall: For a tight set $T$ of parameter $c$, $|T| = cq$. 
Affine Planes

Definition

The **affine plane** is a linear space with at least three noncollinear points, in which any given point $p$ and line $\ell$ not containing $p$ there is exactly one line $m$ through $p$ which does not meet $\ell$. 
Affine Planes

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**Remark:** We use partial affine planes of \( AG(2, q) \) with \( q^2 \) points, which include half of the lines of a full affine plane.
The Paley Graph in the Affine Plane

\( P(q^2) \) can be represented geometrically in a partial affine plane:

- Two points are on a line in the partial \( AG(2, q) \) if and only if they are adjacent in \( P(q^2) \).

- Consequently, lines in the partial plane are cliques in the graph.
The Paley Graph in the Affine Plane: P(9)

Figure 7: The graph of P(9) and P(9) as a partial affine plane. Each line in this partial affine plane is a clique in the graph.
Describing Tight Sets

**Definition**
A tight set is **indecomposable** if it is not the union of smaller disjoint tight sets.

**Definition**
An **isomorphism class** is a class of tight sets under an edge preserving bijection (their graphs look the same).
The Paley Graph in the Affine Plane: \( P(25) \)

**Figure 8:** Affine \( P(25) \) with a tight set Type II of parameter 2 highlighted
The Paley Graph in the Affine Plane: $P(25)$

Observations:

- There are $\binom{5}{2} \times 3 = 30$ tight sets of parameter 2 which are the union of two disjoint cliques.
The Paley Graph in the Affine Plane: P(25)

Observations:

- There are \( \binom{5}{2} \times 3 = 30 \) tight sets of parameter 2 which are the union of two disjoint cliques.

- There are 100 tight sets of parameter 2 which are \textit{indecomposable}.

- These indecomposable tight sets of parameter 2 in \( P(25) \) are all isomorphic.
# P(25) Tight Set Data Revisited

<table>
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The Paley Graph in the Affine Plane: P(49)

Figure 9: A parameter 2 tight set of P(49) highlighted in the affine plane
The Paley Graph in the Affine Plane: $P(49)$

Figure 10: A staircase parameter 3 tight set of $P(49)$
The Paley Graph in the Affine Plane: $P(49)$

Observations:

- There are $\binom{7}{2} \times 4 = 84$ tight sets of parameter 2 which are the union of 2 cliques (decomposable).
The Paley Graph in the Affine Plane: $P(49)$

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- There are $\binom{7}{2} \times 4 = 84$ tight sets of parameter 2 which are the union of 2 cliques (decomposable).

- There are 588 indecomposable parameter 2 tight sets, all of which are isomorphic.
The Paley Graph in the Affine Plane: $P(49)$

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- For parameter 3, there are 3,668 decomposable and 2,058 indecomposable tight sets.
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- There are 588 indecomposable parameter 2 tight sets, all of which are isomorphic.

- For parameter 3, there are 3,668 decomposable and 2,058 indecomposable tight sets.

- The 2,058 indecomposable parameter 3 tight sets can be partitioned into 3 classes where all sets within a class are isomorphic.
# P(49) Tight Set Data Revisited

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## P(81) Tight Set Data Revisited

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Surprising Results in $P(121)$ and $P(169)$

- We found computationally that all parameter 2 tight sets in $P(121)$ are a union of 2 cliques. Thus, there are 0 indecomposable tight sets.

- Similarly, in $P(169)$ there are no indecomposable parameter 2 tight sets.
Surprising Results in $P(121)$ and $P(169)$

- We found computationally that all parameter 2 tight sets in $P(121)$ are a union of 2 cliques. Thus, there are 0 indecomposable tight sets.

- Similarly, in $P(169)$ there are no indecomposable parameter 2 tight sets.

- For parameter 3 in $P(121)$, there are 2 isomorphism classes of decomposable tight sets and 10 isomorphism classes of indecomposable tight sets.
Conjectures

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- **Conjecture:** For every $P(q^2)$ where $q$ is prime, there exists an isomorphism class of parameter $\frac{q-1}{2}$ tight sets which follow the “staircase pattern”
Open Questions

- Do all tight sets exhibit either symmetry or a staircase pattern in some parallel class, as we have seen?
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- Do tight sets in $P(q^2)$ always behave differently when $q$ is composite, as we have seen in $P(81)$?
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- Not very many strongly regular graphs have been studied for tight sets outside the context of finite geometry. Do any of the patterns we observed in the Paley graphs generalize?
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