Mathematics of Games - Comps Project Description

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Prisoner's Dilemma

A classic mathematical game is the prisoner's dilemma, in which two prisoners accused of a crime must independently decide whether to confess to the crime to which they were accused. If neither confesses, they will both face a short misdemeanor sentence. If one confesses, he will go free and the other will face a longer felony sentence. If both confess, they will each serve an intermediate sentence. If both do what is strategically in their best interest, then both will confess. This is because, from the point of view of one prisoner, he is always better off if he confesses than if he does not. Unfortunately, this leads to an outcome for both that is worse than if they had each cooperated.

Things become far more complicated, however, when the same two players play the game described above multiple times in a row. This is known as an iterated prisoner's dilemma. Though seemingly simple, there is an abundance of literature concerning optimal strategies, and there have been regular tournaments where researchers submit scripts that compete for the highest score. A simple strategy for the iterated prisoner's dilemma, the tit-for-tat strategy, is described below.

Tit-for-Tat

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Cooperate the first round.
If your opponent defected last round, defect.
If your opponent cooperated last round, cooperate.
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Despite being incredibly simple, this strategy won the first tournament, outperforming a field of vastly more complicated strategies. Since then, small improvements to tit-for-tat have been discovered, but the essence remains the same.

Board Games

Dominion is a recent board game (released in 2008) that has attracted a large amount of interest among board game enthusiasts, such as myself. In Dominion, each player has their own deck of cards which they slowly hone as they play by adding cards to their deck which will help them acquire more powerful cards. Eventually, players hope to acquire Provinces and other cards worth points. At the end of the game, the player with the most points in their deck is the winner. The idea for this project was born when I was reading some discussion of Dominion strategy where someone described the following strategy, dubbed the Big Money strategy.

Big Money

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If you have $8 or more, buy a Province.
If you have $6 – $7, buy a Gold.
If you have $3 – $5, buy a Silver.
If you have $0 – $2, buy nothing.
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What is amazing is that this script, in only four short lines, plays Dominion quite competently, outperforming many more complex strategies, just like tit-for-tat in the iterated prisoner's dilemma above. Indeed, the strategy is so effective that when the game was first released, some players feared this might be the best strategy, ruining the fun of the game. Could that be true? In this project, we will analyze scripts like this in a variety of games.

Project Outline

Here is an outline of some of the things I hope that we will explore during this project. There should be no prerequisites for this project. Some skills that might be helpful (though none are necessary) are a knowledge of any of statistics, economics, programming, algorithms, or games!

- To begin, we'll write our own iterated prisoner's dilemma scripts, and see how they perform against each other. We'll explore some of the relevant literature on the topic, and see what we can reproduce experimentally.
- We'll examine some simpler board games, such as No Thanks and Incan Gold, with no existing literature, and we'll form our own scripts to play these games. We will see what we find!
- Eventually, we will examine a more complex game, Dominion, where there is a large amount of non-academic literature on strategy. In addition to what we will learn from our own experiments, there is also a vast database of data from over 21 million online games of Dominion to draw from in our analysis.
- Throughout this project there will be ample opportunities to explore and ask new questions and directions that I have not anticipated. This is strongly encouraged, and will all be part of the fun!