An Age Old Question with an Algebraic Explanation:
"What voting procedure best reflects the overall choice of the voters?"

1 A Brief Survey of Voting Theory

Simply put, voting theory studies how we make decisions, and how we can make the most fair decision. Even as early as the 1200s, philosopher Ramon Llull had ideas of different voting procedures. His ideas were popularized in the 1700s by Nicolas de Condorcet and Jean-Charles, chevalier de Borda. The two fiercely disagreed on the "better" voting procedure. Today their methods are referred to as the Condorcet method (the winner must win in every head to head election) and the Borda count (the winner is the one with the most points where points are assigned to a voter's preference in a linearly decreasing order). More recently in the 1900s, Donald Saari has taken a geometric perspective to explain paradoxes that can occur in voting theory. In the last few years, Michael Orrison and his coauthors have taken a more algebraic perspective to explain some of the phenomena that occurs in voting theory. In particular, he uses the tools of the representation theory of the symmetric group to prove results in 'Voting, the Symmetric Group, and Representation Theory' (see link below).

2 A Strange Example

Imagine a scenario where three candidates, called 1, 2 and 3, are up for election. The voters must give a ranking of candidates. Below, when we write $i > j > k$, we mean candidate i beats candidate j, who beats candidate k and $i = j$ represents candidates i and j ranked equally. In the table, we give the outcome of 11 voters picking their candidate preferences.

	$1 \geq 2 \geq 3$	$1 \geq 3 \geq 2$	$2 \geq 1 \geq 3$	$2 \geq 3 \geq 1$	$3 \geq 1 \geq 2$	$3 \geq 2 \geq 1$
No. of votes for each preference	3	2	0	2	0	4

Voting Procedure 1: 1st place gets one point; while 2nd and 3rd place get zero points.
Voting Procedure 2: 1st and 2nd place get one point; while 3rd place gets zero points.

	Candidate 1	Candidate 2	Candidate 3
Total Weights (Procedure 1)	5	2	4
Total Weights (Procedure 2)	5	9	8

So without changing the voter's preference we have two different election outcomes!!!

3 Project Description

Our main task will be to read the paper by Michael Orrison ('Voting, the Symmetric Group, and Representation Theory'). The following will be among the outcomes of our reading the paper.

1. Learning about combinatorial objects called tabloids.
2. Learning some basic representation theory of the symmetric group.
3. Applying the tools we've developed to prove things like: the voting procedure determines the outcome of an election, not the voter's preference.
4. We will also take a look at the Borda count versus the Condorcet method.

4 More Information

- Prerequisites: Math 342 (Abstract Algebra I)
- Faculty Advisor: Peri Shereen
- Project Terms: Winter and Spring
- Find 'Voting, the Symmetric Group, and Representation Theory' at `http://arxiv.org/pdf/0712.2837v1.pdf`