The ABCDs and EFGs of classifying Lie algebras over $\mathbb{C}$

1 | Information
- Supervisor: Peri Shereen
- Terms: Fall and Winter of 2016-2017
- Prerequisite: Math 332 (Advanced Linear Algebra)
- Reading: Introductions to Lie Algebras by Karin Erdmann and Mark J. Wildon

2 | An Example

You have probably come across a Lie algebra at some point in your mathematical education. First, a Lie algebra is a vector space. So, for example, $\mathbb{R}^3$ is a vector space where our scalars are over the real numbers. Second, a Lie algebra has a product operation under which it is closed. In $\mathbb{R}^3$, if we take the cross product of two vectors we get yet another vector. Third, the product operation must satisfy three nice properties, which the cross product happens to satisfy. Therefore, the vector space $\mathbb{R}^3$ over $\mathbb{R}$ with the cross product defines a Lie algebra.

3 | Description

Lie theory arose out of the work by Sophus Lie in the late 1800s. At the time Lie was concerned with using group theory to solve ordinary differential equations. The mathematics that arose out of his quest led into what is today called Lie groups. One can ‘linearize’ a Lie group to get a Lie algebra. A common mathematical question is: does there exist a classification of our object of study? Much work has been done since Sophus Lie in classifying finite dimensional Lie algebras over $\mathbb{C}$.

We will begin the comps project by learning about Lie algebras and some main results in the discipline. The main objective will be to explore the classification result of the finite dimensional simple Lie algebras over $\mathbb{C}$. In the process we will discover important Lie algebra tools like: the Weyl group (which is a Coexter group), Cartan matrices, Root Systems and Dynkin Diagrams. All of these tools encode much of the Lie theoretic structure of each type.