Knot Theory

What is a mathematical knot? Any five year old will happily tangle up one of your shoelaces into a real mess. A mathematical knot, though, is somewhat even nastier: take the messed up shoelace and then fuse the two ends of the shoelace together. If the five year old did a sufficiently crazy job of it, then there’s no way to un-knot the fused-together shoelace no matter how much you push this strand under and pull that strand over another. Now, can you prove that there’s no way to un-knot it? What if you were only allowed to look at it (no touching!)?

More technically, a knot is an equivalence class of smooth functions $f : S^1 \rightarrow \mathbb{R}^3$, where $S^1$ denotes the unit circle in $\mathbb{R}^2$ and where two functions are equivalent if they are related by some allowed moves that capture our intuitive idea of “push this strand under and pull that strand over”. In general, knot theorists try to develop invariants to distinguish two knots apart, with the ultimate goal of making a list of all possible knots. The techniques involve many different branches of mathematics— anywhere from number theory to hyperbolic geometry. More surprisingly, knot theorists often draw inspiration from and are motivated by questions in other sciences. For instance, a recently discovered and powerful knot invariant called the Jones polynomial was motivated by quantum physics, and questions about knotted molecules like DNA and certain proteins often open up new research possibilities.

Knot theory is a rich field of study and offers plenty of opportunities to explore new and different kinds of math. So there is quite some room for the comps project to evolve to suit student interest.

In previous years, I’ve led comps groups studying the Jones polynomial for knots that are contained in a thickened surface, as well as a comps group studying the effect of families of topoisomerases on DNA knotting. Other possibilities might include: understanding how knots are related to quantum computing or to hyperbolic 3-manifolds, investigating the ways in which proteins are knotted, or finding a new and better way to quantify the knottedness of a non-fused-together shoelace.

The expected number of students in this comps project will be somewhere between 8 and 12. However, we will try to retain as much of the flavor of a small group comps as is possible. The first part consists of a 395 seminar course in Winter 2014, where we will begin by reviewing the basic theory of knots and survey some important knot invariants. Together as a large group, we will identify some specific problems for further investigation. The second part will see students split into smaller groups of 3 to 4 students to focus on particular areas of student interest. The comps project will be more fluid at this stage; it might involve further reading in journal articles and graduate texts or research on an open problem. Although we will still meet as a large group, I expect to meet the small group more regularly during the second part of the comps project.

The requirement for this comps is Topology (Math 354) or Abstract Algebra (Math 342) or permission of the instructor.