Trojan Y-Chromosome Approach to Dealing with Invasive Species

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19 July 2013

Funded by the National Science Foundation and Texas A&M
The Problem
Solutions

- Poison
- Introduction of Foreign Species
Trojan Breeding
Basic Model

\[ \frac{df}{dt} = \frac{1}{2} \beta fmL - \delta f \]
\[ \frac{dm}{dt} = \left( \frac{1}{2} fm + \frac{1}{2} rm + fs \right) \beta L - \delta m \]
\[ \frac{ds}{dt} = \left( \frac{1}{2} rm + rs \right) \beta L - \delta s \]
\[ \frac{dr}{dt} = \mu - \delta r \]
\[ L = 1 - \frac{f + m + s + r}{K} \]
Proportional Model

\[
\frac{df}{dt} = \frac{1}{2} \beta fmP_{fm}L - \delta f
\]
\[
\frac{dm}{dt} = \left( \frac{1}{2} fmP_{fm} + \frac{1}{2} rmP_{rm} + fsP_{fs} \right) \beta L - \delta m
\]
\[
\frac{ds}{dt} = \left( \frac{1}{2} rmP_{rm} + rsP_{rs} \right) \beta L - \delta s
\]
\[
\frac{dr}{dt} = \mu - \delta r
\]
\[
L = 1 - \frac{f+m+s+r}{K}
\]

\[
P_{fm} = \frac{fm}{(m+s)(f+r)}
\]
\[
P_{fs} = \frac{fs}{(m+s)(f+r)}
\]
\[
P_{rm} = \frac{rm}{(m+s)(f+r)}
\]
\[
P_{rs} = \frac{rs}{(m+s)(f+r)}
\]

Bang–Bang Proportional Analysis (F4) for B = 1, d = 0.5, K = 100, \(\mu = 14\)
μ Minimization

Comparative Mu–Space Bang–Bang Analysis (F4) for B = 1, d = .5, K = 100

- Basic Model
- Proportional Model

Total Trojans Introduced

μu
Stochasticity

- Low Population Systems
- Random Events
0-Dimensional System

\[ F \text{ Birth } = \frac{1}{2} \beta fm \]
\[ M \text{ Birth } = \beta \left( \frac{1}{2} fm + \frac{1}{2} rm + fs \right) \]
\[ S \text{ Birth } = \beta \left( \frac{1}{2} rm + rs \right) \]
\[ R \text{ Birth } = \mu \]
\[ L' = \frac{f+m+s+r}{K} \]

\[ F \text{ Death } = \frac{1}{2} \beta fmL' + f \]
\[ M \text{ Death } = \beta L' \left( \frac{1}{2} fm + \frac{1}{2} rm + fs \right) + m \]
\[ S \text{ Death } = \beta L' \left( \frac{1}{2} rm + rs \right) + s \]
\[ R \text{ Death } = r \]
Normalized Probability Mass Function for Wild-Type Female Extinction
For mean = 2.4208, variance = 3.9316, and skewness = 12.1638
2-dimensional System

\[ F \text{ Birth} = \frac{1}{2} \beta fm \]
\[ M \text{ Birth} = \beta \left( \frac{1}{2} fm + \frac{1}{2} rm + fs \right) \]
\[ S \text{ Birth} = \beta \left( \frac{1}{2} rm + rs \right) \]
\[ R \text{ Birth} = \mu \]
\[ L' = \frac{f+m+s+r}{K} \]
\[ F \text{ Death} = \frac{1}{2} \beta fm L' + f \]
\[ M \text{ Death} = \beta L' \left( \frac{1}{2} fm + \frac{1}{2} rm + fs \right) + m \]
\[ S \text{ Death} = \beta L' \left( \frac{1}{2} rm + rs \right) + s \]
\[ R \text{ Death} = r \]

\[ \text{Migrate} = \frac{\text{population} \ast \text{migration-speed}}{\text{cell-length}} \]
• mean = 35.3 (tδ)
• variance = 465
• skewness = 11,500
• mean = 28.8 (tδ)
• variance = 295
• skewness = 6690

Max time = 26.7