Seminar in Probability

Number of students: 8-12

Topics for this comps will be taken from senior undergraduate and graduate level topics in probability, and will be based on the background and interests of the students. Three areas of possible focus include:

Coupling and Poisson approximation. Coupling is a powerful purely probabilistic technique for obtaining bounds and approximations for random variables. The Poisson approximation of the Binomial distribution is a classic result for i.i.d. random variables. Less known is that Poisson approximation can be used for dependent variables as well, particularly in settings involving counts of combinatorial objects.

Martingales and Conditional Expectation. Martingales are generalizations of fair games. A sequence of random variables $X_1, X_2, \ldots$, is a martingale if $E[X_{n+1}|X_n, \ldots, X_1] = X_n$ for all n. Knowledge of the past gives no information about the expected value of the future. Unbiased random walks are examples of martingales. The optional stopping theorem, a major theorem in probability, states that under certain conditions the expected value of a martingale at a random stopping time is equal to its initial value. This has been used to prove, for instance, the impossibility of a successful betting strategy to beat the house.

Probability and Measure. The theory of probability taught in a first course like 265 works great for discrete probability spaces. But problems involving infinite sequences of coin tosses, the random drawing of a point from an interval, and limits of random variables require measure theory to put probability on a fully rigorous foundation.

This comps project will begin as a seminar-style class. After treating the core topics, students will read current papers and research articles in their areas of interest. As the comps evolves, students will present their readings, seek out material on their own and hopefully find related problems to research.

The requirement for this comps is Math 265 (or Probability in Budapest) Some exposure to Real Analysis, Measure Theory, and/or Combinatorics would be great, but not required. This is a winter/spring project and Math 265 will be offered in the fall, so if you havent taken Probability yet you can still choose this project, as long as you plan to take Math 265 (or Probability in Budapest) in the fall.