Winter Term Course Descriptions

Math 206: A Tour of Mathematics
(1 credit; S/Cr/NC)
Instructors: Much of the Math Department
Time: Fridays only, 6a (3:30-4:30)
Prerequisite: None
Are you considering a math major, but wondering just what will follow after all the calculus and linear algebra, or where the frontiers of mathematical knowledge are to be found? Are you already a major who would enjoy some fresh perspectives on, and new insights into, your chosen subject? Come join us for a series of lectures on a variety of mathematical topics, with emphasis on exciting ideas, concepts and results rather than on systematic coverage of any particular subject (we have other courses for that). Although this course has been offered yearly, there should be no overlap in lectures with last winter’s offering, so you can “repeat” once if you took the 2009 Tour.

Math 236: Mathematical Structures
Instructor: Mark Krusemeyer
Time: 5a
Prerequisite: Math 232 or instructor’s consent
How do we prove mathematical results? How do we even think of possible results, and what makes us suspect that a particular result may be true? There are no easy, general answers. Mathematics is a complex subject, with a great variety of living and growing branches, and with deep roots that tap into the wisdom of many generations. Nevertheless, if you’ve ever wondered “How could anyone come up with that?”, or “How can you be really sure of that?”, about some piece of mathematics, taking this course may help dispel some of the mystery. We’ll explore various concepts, especially from set theory, that are indispensable for most areas of advanced mathematics, and we’ll spend considerable time developing theorem-proving and problem-solving skills. Along the way we’ll take a new and closer look at some old friends, such as functions and relations: What are they really? If you’re considering a math major, taking this course should help you decide; also, “Structures” is a prerequisite for most upper-level math courses.

Math 241: Ordinary Differential Equations
Instructor: Mark Krusemeyer
Time: 3a
Prerequisite: Math 232 or instructor’s consent
In calculus you may well study separable first-order differential equations for a bit, but that’s just the tip of the iceberg! In any field where mathematics is applied, you are likely to find equations relating unknown functions and their derivatives. Over the centuries, following the lead of Newton, Leibniz, and the Bernoullis, mathematicians have come to grips with many such equations. Naturally, they prefer to get exact solutions if possible, and we’ll look at some of the systematic methods (and a few of the clever ad hoc tricks) that have been developed to find solutions. On the other hand, there are times when finding an exact solution is too difficult, or even potentially misleading - for instance, because the mathematical model that leads to the differential equation is imprecise to begin with. In such cases, it is often best to concentrate on the qualitative behavior of solutions; for example, you might try to predict what will happen in the long run.

In this course, you’ll find plenty of calculus-style computation, including ample opportunity to brush up on your techniques of integration (Mathematica can help with some of that), but also a few theoretical discussions, some geometric ideas, and a bit of mathematical modeling.
The textbook, which was written by a close (younger!) relative, does not presuppose much linear algebra, but concepts from linear algebra, ranging from vector spaces of functions through linear transformations and kernels to eigenvalues and eigenvectors, will be mentioned and used with some regularity in class.

Math 245: Applied Regression Analysis  
Instructor: Bob Dobrow  
Time: 2a  
Prerequisite: Math 215 or Math 275  
Regression is the meat and potatoes of statistics. If you’ve taken a first stats class, you can enjoy a delicious second serving in this very applied, very “hands on,” very computational course. Some applications you might find particularly appetizing are: how Google uses logistic regression to model flu trends, how exit polls can predict an election, regression models for global climate change, what the variables, age, race, and gender can tell us about how people react to medication. We will cover several types of regression models with an emphasis on model building and checking. The class will meet in the stats lab with use of the statistical freeware package R. This class is designated as sophomore priority. (Math 245 will also be offered in the Spring.)

Math 275: Introduction to Statistical Inference  
Instructor: Katie St. Clair  
Time: 3a  
Prerequisite: Math 265  
Statistical Inference is the process of drawing conclusions from data. In Math 265 you learned how to compute probabilities. Statistics starts with the tools of probability and calculus and applies them to real life problems. In the process, we will leave the “pure” math classroom, where questions are well-posed and solutions are right or wrong, and plunge into the reality that uncertainty is everywhere and data are dirty. In this class, we’ll look at numerous applications and case studies drawn from the environment, biology and medicine, and social science. The statistical software languages S-PLUS and R will be introduced and used throughout.

Math 342: Abstract Algebra I  
Instructor: Eric Egge  
Time: 2a  
Prerequisite: Math 236  
Abstract algebra (not to be confused with elementary algebra, which you studied in high school or before) is a pillar of pure mathematics which supports a large body of work both within and outside of mathematics. Although ideas from abstract algebra are often studied for their intrinsic interest, they have also been applied in theoretical physics, in the design of error-correcting codes, in quantum chemistry, and even in the study of symmetry and artistic patterns like those which appear in Escher’s “Regular Division of the Plane” drawings. In this course we will study groups, rings, and fields, which are generalizations of ordinary arithmetic systems like “clock arithmetic”, sets of matrices, and the set of real numbers, in the same way that abstract vector spaces generalize $\mathbb{R}^n$. If time permits we will see how some famous construction problems from Greek geometry were proved to be unsolvable, about 2000 years after they were first posed!

Math 354: Topology  
Instructor: Helen Wong  
Time: 4a  
Prerequisite: Math 236  
The oldest (in more sense than one!) topology joke: A topologist is a person who doesn’t know the difference between a coffee cup and a donut. In this course, we’ll look at the mathematics underpinning the intuition. What is a topological space and what kinds of deformation are allowable? For instance, we might want to keep the donut-ness of a donut and not allow it to be squished into one giant doughball. We’ll build definitions from the ground up, ending with some modern algebraic techniques for distinguishing topological spaces apart.
Math 395: Functional Analysis
Instructor: Jon Armel
Time: 5a
Prerequisite: Math 321 or instructor’s consent

Functional analysis is linear algebra in infinite-dimensional vector spaces, usually spaces of functions. When studying linear algebra in finite dimensions, there is really only one reasonable topology, and all linear transformations are continuous. An infinite dimensional vector space, however, may admit many different topologies, leading to different notions of convergence and continuity.

As we investigate these concepts, we will move away from the idea of a function as a correspondence between two sets. You may have already seen that two functions are identified if they agree almost everywhere, so their values at a particular point are of little importance. We will continue this as we first consider how functions behave as simply points in a vector space. Toward the end of the course, we will further generalize functions in a way that will allow us to take the derivative of anything you could ever dream of (discontinuous functions, measures, differential operators, and more).

CS 252: Algorithms
Instructor: David Liben-Nowell
Time: 2a
Prerequisites: CS 201; CS 202 or Math 236

What with Halloween here and all, you start to wonder: suppose that, after you don your Rob Oden costume and head out into the night, you want to hit all of the houses of Carleton’s political science professors while you’re trick-or-treating. (They have the best candy.) But you’re feeling lazy, so you want to be sure that your route is as efficient as possible. So you decide to write a program to solve this problem. How would you go about designing a solution for this problem? How fast can your program be? Sadly, some problems are just plain hard. How do you solve the easy problems? How do you identify the hard problems? How do you deal with a hard problem when you’re confronted with it? CS 252, which counts toward both mathematics and computer science majors, is concerned with developing techniques for the design and analysis of algorithms. We will cover several major algorithm design techniques, computational complexity focusing on NP-completeness, and algorithmic techniques for intractable problems.

Abstract Deadline for JMM 2010

The registration deadline for participating in the Undergraduate Student Poster Session at the Joint Math Meetings January 13-16, 2010, in San Francisco is November 6. To submit an abstract, go to www.maa.org/students/undergrad/jmmposterindex.html. Those who register for the Poster Session will have the opportunity to apply for a travel grant up to $600 through the MAA and possibly other funds through our department.

Mr. Gardner turns 95

Martin Gardner has been entertaining us with his recreational mathematics for the past fifty-three years. At first, Gardner was hesitant to write for the Scientific American because he had never taken a mathematics course beyond high school and struggled with calculus even then. He took the gamble however, and at 95 years old, he is still providing us with entertaining mathematical puzzles. To further celebrate his birthday, Gardner published his 70th book of essays and puzzles. Let’s help him celebrate by solving one of his recent riddles:

Five toothpicks form the giraffe shown above. Change the position of just one pick and leave the giraffe in exactly the same form as before. The re-formed animal may alter its orientation or be mirror reversed but must have its pattern unchanged. Answer in next week’s gazette.
Careers of Carleton Math Alumni

Are you a math major and wondering what you could be doing five to ten years after graduation? Did you know that 31% of our math majors earn a master’s degree, an additional 16% go on to finish a PhD, and another 12% earn an MBA, JD or MD? Have you seen the pie charts in our 2nd floor hallway of the CMC? Stop and visit them to see what our math major alumni are doing (and to be able to read the font)!

The Calculus of Friendship

Have a little extra time this weekend? Check out Steven Strogatz’s new book, The Calculus of Friendship. Documenting over thirty years of mathematics-centered letters between Strogatz and his high school calculus teacher, the book illustrates what a teacher and student learned about life.

Problems of the Week

1. Find all real functions $f$ such that, for every real number $x$, $f(x + 2\pi) = f(x)$ and $f'(x) = f(x + \pi) - 2\pi$.

2. For each positive integer $n$, let $Z(n)$ be the number of 0’s to the right of $n!$. For example, $Z(6) = 1$ because $6! = 720$, while $Z(11) = 2$ because $11! = 39,916,800$. Find, with justification,

$$\lim_{n \to \infty} \frac{Z(n)}{n}$$

Henry Luo, Li Shunji and “Drizzle Hater” each solved both of last week’s problems. The lottery winner this week is Henry Luo. He should stop by CMC 217 to collect a prize from the B.B.O.P. As always, to earn a chance to visit the B.B.O.P. you must submit a correct solution to one or both of the above problems by Tuesday night.

If you want to take a break from problem solving, you can check out solutions to earlier problems that are posted in the hallway by the 2nd floor offices.

Gail Nelson

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