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Do you want to major in Math or Statistics?

Thinking about being a math or statistics major? Come learn about our department and courses. Hear about what fellow Carls have gone on to do with a degree in mathematics or statistics. Speak with current majors. Cookies will be served! The event will take place at 4:00 pm on Tuesday, February 14 in CMC 206, so mark your calendars.

Individual Comps Talks

The students who chose to present their independent comps during Winter Term are about to put the finishing touches on their presentations. They'll present in CMC 206 beginning at 3:00 pm on Thursday, February 16. Stop by for a talk or two (or all of them!) and you'll be sure to learn something about the fields of mathematics and statistics you've never encountered!

Title: Assignment Problem
Speaker: Shatian Wang
Time: 3:00 - 3:30
Combinatorial optimization is an important research area in applied mathematics, theoretical computer science and operations research. It is concerned with finding an optimal object from a finite, discrete set of objects, and has crucial applications to the fields of machine learning, artificial intelligence, software engineering, and others. The assignment problem is a classical combinatorial optimization problem: given n jobs, n machines, and the cost of performing job i on machine j for each job-machine pair, our goal is to find an optimal assignment of the n jobs to the n machines such that the total cost is minimized. This talk will first formulate the static assignment problem as a linear program, and introduce the simplex method for solving general linear programs. The stochastic version of the assignment problem is then discussed, with an important probability theorem that gives an upper bound to the expected cost of the optimal assignment.

Title: Penalized Spline Regression
Speaker: Josh You
Time: 3:30 - 4:00
Penalized splines are non-parametric regression models involving compositions of piecewise functions that can provide a closer fit to data sets which simpler parametric regression models may fail to model. This talk will discuss how to create penalized spline models by applying a roughness penalty when finding the ordinary least-squares fit. We will also discuss the different types of penalized splines and how to choose the appropriate model type and parameters, and observe splines in action with example data sets.

Title: Hyperreal Numbers and Nonstandard Analysis
Speaker: Isaac Garfinkle
Time: 4:00 - 4:30
The hyperreal numbers are an extension of the reals that include infinite and infinitesimal quantities. If we extend real sequences and functions to the hyperreals, properties like convergence and continuity become far more intuitive. For example, the derivative of a function can be thought of as the slope of a line passing through two points that are infinitely close together on the curve. We will develop the hyperreals and their relationship to the standard reals. We will then use the hyperreals to more easily prove some familiar results about the real numbers.

Title: Using Statistical and Computational Techniques to Characterize Deformation in Geologic Systems
Speaker: Sam Bacon
Time: 4:30 - 5:00

Deciphering the deformational history of a region is a common goal for structural geologists. Formulating an accurate history, however, is very difficult given the nature of the available observations. How can a geologist determine how good their guess is? How do they make the best one? In this talk we will discuss topics ranging from fluid dynamics, the finite element method, Lie theory, structural geology, statistical inference and orientational statistics to explain how to obtain a maximum-likelihood estimate of a deformation history, given common field observations. In particular, we explore the use of these techniques on an idealized mid-ocean ridge-transform system.

Title: Expectations of Life and Exploratory Factor Analysis
Speaker: Olivia He
Time: 5:00 - 5:30
How do we study concepts that cannot be measured directly? How do we simplify multivariate data to uncover the latent structures? Exploratory factor analysis is a commonly used method to achieve these goals. Sociologists and psychologists often use this method to study concepts that cannot be directly observed, such as social class and intelligence. In this talk I will explain the purpose of exploratory factor analysis and the mathematical theories and assumptions behind it. I will also introduce different ways of estimating parameters in exploratory factor analysis and how to interpret the results. Additionally, I will use an example on expectations of life to illustrate the use of exploratory factor analysis. Limitations and criticisms on this method will also be discussed.

Title: Item Response Theory
Speaker: Caroline Duke
Time: 5:30 - 6:00
Item response theory is a revolutionary test theory that is used to create and analyze surveys, tests, and questionnaires. The theory is used to create effective testing tools for a wide range of settings, including hospitals, educational institutions, companies, and others. For example, it is used to create the LSAT exam, personality tests, patient outcome surveys, and customer satisfaction surveys, just to name a few. During this presentation, you will learn about how item response theory works and how it is applied to many different fields of study.

Title: Building a Dynamical System: with Voronoi Tessellations
Speaker: Samantha Sheldon
Time: 6:00 - 6:30
In this talk, we will explore a dynamical system based on the vertices of Vorono tessellations. A Voronoi tessellation is a partitioning of a plane with a set of points into convex polygons that are defined based on their distance to a generating point. We will learn how this system is constructed and discover the upper and lower bounds on its size. Finally, we will look at infinite configurations of points that are periodic.

What is the Math and Stats Department Offering Next Term?

Have you checked your registration number yet? Made a list of classes you're hoping to take next term? Let the course descriptions below guide you into an adventerous spring term within the Carleton Department of Mathematics and Statistics! There's something for everybody, from statistics to combinatoral theory and from differential equations (which may be called ordinary but are, in fact, truly neat) to a seminar about the theory of elliptic curves-- find out more below.

Math 236: Mathematical Structures
Instructor: Mark Krusemeyer, Helen Wong
Time: 3a, 4a
Prerequisite: Math 232 or permission of the instructor
How do we prove mathematical statements? How do we even think of possible statements, and what makes us suspect that a particular statement may be true? There are no easy, general answers. Mathematics is a complex subject, with a great variety of living and growing branches, and with deep roots that tap into the wisdom of many generations. Nevertheless, if you've ever wondered "How could anyone come up with that?", or "How can you be really sure of that?", about some mathematical result, taking this course may help dispel some of the mystery. We'll explore various concepts, especially from set theory, that are indispensable for most areas of advanced mathematics, and we'll spend considerable time developing theorem-proving and problem-solving skills. Along the way we'll take a new and closer look at some old friends, such as functions and relations: What are they really? If you're considering a math major, taking this course should help you decide; also,"Structures"Â is a prerequisite for the majority of upper-level math courses.

Math 241: Ordinary Differential Equations
Instructor: Sam Patterson
Time: 5a
Prerequisite: Math 232 or permission of the instructor
The language and tools of differential equations are used by mathematicians and scientists to describe and understand the world. In this course we will study ordinary differential equations from both a practical and theoretical point of view. We will see how mathematical models are developed from natural laws and used to study physical systems, including classical examples such as falling objects, mass-spring systems, pendulums, and predator-prey models. Using ideas from calculus and linear algebra, we will develop methods for solving differential equations when we can, and learn how gain understanding even when we can't. In fact, most differential equations cannot be solved explicitly in terms of known functions, so we will develop techniques for qualitative analysis and numerical approximation. We will learn how mathematicians approach ordinary differential equations using a combination of theory, computer computation, and computations by hand.

Math 245: Applied Regression Analysis
Instructor: Katie St. Clair
Time: 2a
Prerequisite: Math 215 (or equivalent) or Math 275
Model building is a fundamental idea in statistics. In your intro stats class you learned some basic techniques for modeling a response as a linear function of one explanatory variable (simple linear regression). In this second stats course you will learn more advanced techniques for building regression models that can include many explanatory variables (multiple regression) or a categorical response (logistic regression). We will apply these techniques to explore how air pollutants might affect mortality, whether sex plays a role in determining a worker's salary, and how a regression model predicted a national tragedy. This course emphasizes model building and checking techniques and statistical writing. We will meet in a lab and use the free statistical software R. As the title suggests, this is an applied course so you will be working with new data sets each week, and you can expect to be a seasoned R user by the end of the term!

Math 261: Functions of a Complex Variable
Instructor: Sam Patterson
Time: 3a
Prerequisite: Math 211

The premise of this course is simple--not at all complex. In Calculus, you explored concepts of limit, derivative and integral of a certain kind of function: real-valued functions of a real variable i.e. real number in -- real number out. In Math 261 we play the same game but with complex-valued functions of a complex variable. This may sound ho-hum but the results will astound you. This seemingly modest exploration will lead us to some of the most beautiful results in mathematics. But, these results are not only beautiful but practical. What started as an abstract game for mathematicians has evolved into an essential toolkit for scientists and engineers. No doubt the subject of complex variables is, in part, what physicist Eugene Wigner was referring to when he spoke of "The unreasonable effectiveness of mathematics in the natural sciences."Â This course, Math 261, will focus on these effective tools, whereas Math 361 takes a more theoretical approach to the same subject.

Math 275: Introduction to Statistical Inference
Instructor: Andy Poppick
Time: 5a
Prerequisite: Math 265

Statistics is the discipline concerned with how data are used to understand uncertainty in populations or processes (physical, social, etc.) exhibiting inherent variability. In this course, we develop tools to evaluate what we know - and what we don't know - about the observed world. We will introduce inferential methods at a level that uses the basic language of probability and we'll learn to apply these methods in realistic settings to answer interesting questions. An additional emphasis will be placed on computational tools for data analysis, using R.

Math 280: Statistical Consulting
Instructor: Katie St. Clair
Time: Thursdays only, 2/3c
Prerequisite: Math 245

Students will work on data analysis projects solicited from the local community. We will also cover the fundamentals of being a statistical consultant, including matters of professionalism, ethics and communication.

Math 295: Coding Theory
Instructor: Peri Shereen
Time: 2a
Prerequisite: Math 236

Imagine you are sending data over noisy airwaves. Your friend on the other side receives the data you sent, but some distortion occurred along the way. How can your friend determine what the original data was? Answering this question will be the premise of what we study. To answer the question we will use algebraic tools. Some of the algebra you may know from linear algebra, and some algebra we will discover along the way.

Math 331: Real Analysis II
Instructor: Liz Sattler
Time: 2/3c
Prerequisite: Math 321

You saw calculus once, and then you really saw calculus again in the form of Real Analysis 1. What other analysis secrets are we hiding? In this course, we will investigate the integral. By now, we have a solid understanding that a definite integral (as we saw in previous courses) calculates the area under the curve. We learned how to estimate the area that using Riemann sums, but is this the only way to compute the area under a curve? To fully answer this question, we will learn about Lebesgue measure, the Lebesgue integral, general measure theory (with a possible special appearance from fractal measures), and L^p spaces. This course includes topics that appear in many first-year graduate analysis courses. It is highly recommended for anyone who is seriously considering graduate school in math or a related field.

Math 333: Combinatorial Theory
Instructor: Eric Egge
Time: 3a
Prerequisite: Math 333 or a course in Enumerative Combinatorics in Hungary or permission of the instructor
I looked in my sock drawer this morning and saw a jumbled collection of 36 socks, consisting of 6 socks in each of 6 colors. Each sock had a single letter stitched on it, and within each color, each of the letters J, S, E, A, K, and R appeared exactly once. In how many ways, I wondered, can I match my 36 socks into 18 pairs, so that both socks in each pair have the same color? When spring comes (it's just 12 weeks away), and I don't need socks any more, can I display my socks in a 6 by 6 square, with no color or letter repeated in any row or column? Then my son wandered into the room. He has his own definition of which pairs of socks match, which seems to have nothing to do with letters or colors. If I close my eyes and start removing socks from the drawer, how many must I remove before I am guaranteed to have three socks in which each pair matches (according to my son) or three socks in which no pair matches?

If you, like me, are intrigued (or tormented) by questions like these, then combinatorics might be the right course for you. We'll study techniques for showing certain arrangements of things exist (or don't), and techniques for counting these arrangements when they do exist. Some of these counting techniques involve playing with power series, without worrying about convergence! We'll pay particular attention to counting sequences (like the Catalan numbers and the partition numbers) which have especially remarkable properties, and we'll use our counting techniques to prove some of the myriad identities involving the numbers in Pascal's triangle. We'll also make periodic forays into graph theory, and near the end of the course we'll see a "proof" of the four color theorem. Although this proof will have a gap (which I'll ask you to find), we will also learn about the key ideas in the actual proof of this famous result.

You don't need any previous knowledge of combinatorics to take this course, just experience with the proof techniques from structures, the ability to multiply polynomials, a willingness to try new and strange problems, and a sense of adventure.

Math 341: Fourier Series and Boundary Value Problems
Instructor: Rob Thompson
Time: 5a
Prerequisite: Math 241

The ideas of Fourier analysis are ubiquitous in modern math, and surprisingly ancient. Nearly 2000 years ago, believing the earth to be the center of the universe, Ptolemy explained the erratic motion of the sun and other planets through a theory of epicycles; all bodies moved around the earth by tracing out paths along nested spheres in the heavens. More spheres were needed to explain more complicated motion. Search youtube for "Ptolemy and Homer (Simpson)"Â to see an amusing example.

The connection with pde came after the invention of calculus. About 200 years ago, Jean Baptiste Fourier studied the way that heat moves through a flat metal plate via a pde called the heat equation. Trying to describe his observations mathematically, he did essentially the same thing as Ptolemy: he expressed a heat distribution as a sum of sines and cosines (motion along nested circles/spheres, in a sense). Expressing the complicated behavior of heat in terms of simpler functions gave Fourier powerful insight into the behavior of the heat equation.

In this course we'll study various interesting pde (including the heat equation) and their applications to wave propagation, heat conduction, elastic equilibrium, quantum particle motion, and more. We'll also develop ideas from Fourier analysis as needed to access information about the solutions to the pde we study. Feel free to contact me (rthompson) with any questions!

Math 349: Methods of Teaching Mathematics
Instructor: Steve Kennedy
Time: 2/3c
Prerequisite: Junior or Senior Standing

How is mathematics taught? You've certainly seen mathematics taught, and if you're a tutor or have a friend in a lower-level math class, you've probably done some teaching. Is there a best way to teach? How do students learn mathematics? What is a lesson plan? What's important when you're in front of a class? Through readings and observations and practice, we'll discuss these questions and you'll develop your own answers. Enrollment in this course requires a time commitment outside of class observing in the Northfield public schools.

Math 365: Stochastic Processes
Instructor: Bob Dobrow
Time: 4a
Prerequisite: Math 232 and 265

A stochastic process, also known as a random process, is a model for systems that evolve in time and/or space and for which there is inherent uncertainty. They are used to model applications as diverse as the evolution of DNA sequences, the spread of infectious diseases, the vicissitudes of the stock market, the diffusion of gases, tomorrow's weather, and your best stopping strategy at the casino.

The formal definition is rather stale: a stochastic process is a collection of random variables defined on a common probability space. An independent sequence is a simple -- and boring -- example. Things get interesting when you allow some dependency among the random variables giving rise to a rich class of objects like Markov chains, random walk, Poisson processes, and Brownian motion.

This course is for those who like probability. Important tools include conditioning and conditional expectation, with doses of graph theory, combinatorics, and analysis. Markov chains, a central topic of the course, bring together probability and linear algebra in remarkable ways. Some fun topics we will explore include: how long, on average, it takes to play Chutes and Ladders, how randomization can be used to decode messages, how continuous nowhere-differentiable functions are used to model stock prices, and what the eigenvalues of a Markov matrix reveal about how many shuffles it takes to mix up a deck of cards.

The course is preparation for students interested in the use of probability in scientific applications, as well as statistics, actuarial studies, and advanced probability. There will be some use of technology, in particular working simulations in R, but no prior computing knowledge is assumed. Students will write up many assignments in LaTeX.

Math 395: Topics in the Theory of Elliptic Curves
Instructor: Rafe Jones
Time: 4a
Prerequisite: Math 342 or equivalent in Budapest Study Abroad Program or permission of the instructor

Elliptic curves lie at the intersection of geometry, algebra, and number theory, and provide a beautiful interplay among major ideas in each of these disciplines. An elliptic curve is at the same time a curve -- similar in some ways to a parabola in the plane -- and also a group, as two points may be "added" in a way that satisfies the group axioms. Thus geometry meets algebra. Points on such a curve with integer or rational coordinates have great significance in number theory, as they furnish solutions to Diophantine problems such as one posed by Fermat in the 1650s: find all positive integers x and y such that the cube of x is two more than the square of y. (Fermat found the solution $x = 3$ and $y = 5$ and it took 150 years to show there are no others.) In this course we will study the geometry of elliptic curves, points of finite order, points with integer coefficients, and points with rational coefficients. The latter part of the course will consist of group presentations on further topics. An enormous amount of current mathematical research involves elliptic curves, and we will discuss some of this ongoing work as we encounter the relevant ideas.

Job & Internship Opportunities

Office Assistant: Math, Stats, and CS Departments

The Mathematics and Statistics Department and Computer Science Department is looking for an Office Assistant for 6 hours per week Spring Term 2017. We are seeing someone who is friendly, reliable, detail-oriented, and have the ability to work independently. Student should be creative and have good written and oral skills. Needing to know Word, Excel, and Publisher. Previous office experience preferred. Responsibilities include general office duties, phone and office coverage, creating spreadsheets and posters, running errands, copying and scanning, and other duties as needed. Students, who are interested should contact Sue Jandro at sjandro@carleton.edu.

Gazette Editor: Math and Stats Department

The Mathematics and Statistics Department is looking for a Gazette Editor for 4 hours per week Spring Term 2017. We are seeking someone who is self-motivated, creative, ability to edit a newsletter, and attention to detail. The Gazette is published every other week and during busy times we may add an additional Gazette. The Gazette editor would need to be available to finalize the newsletter on Thursdays. You will be able to edit the newsletter from outside of the office for some of your hours. Newsletter and editing experience preferred. Students who are interested should contact Sue Jandro at sjandro@carleton.edu.

NRHEG Secondary School: Math Teacher

NRHEG Public Schools is seeking a passionate and innovative teacher for the 2017-2018 school year with the potential of teaching Advanced Algebra, Intermediate Algebra and College in Schools course(s). Candidates should hold a Minnesota teaching license in 7-12th grade math education or the ability to attain one is preferred. For more information and to view the application visit http://nrheg.k12.mn.us/pages/NRHEG.

Problems of the Fortnight

To be acknowledged in the next *Gazette*, solutions to the problems below should reach me by noon on Tuesday, February 21.

1. As a function of n, what is the largest number of 90° angles that an n-gon can have? (For example, the number is 1 for $n = 3$ and 4 for $n = 4$. Note that the n-gon doesn't have to be convex; in other words, it can have "indentations". On the other hand, 270° angles *don't* count as 90°. For instance, if you make a 6-gon by drawing the outline of a "fat" letter L, it will have five, but not six, 90° angles.)

2. Let $y = f(x)$ be a polynomial of degree $d > 1$. For various points P in the plane, consider the points Q on the graph of the polynomial such that the tangent line at Q to the graph passes through P.

   a) Show that if d is odd, then for every point P there will be at least one such point Q (that is, there will be at least one tangent line to the graph that passes through P).

   b) Suppose d is fixed (not necessarily odd) but we allow any polynomial of degree d and any point P in the plane. What is the largest number of points Q that is possible? Note that to solve this completely, you should show why you can indeed get that number of different points Q, as well as that you cannot get any more.

Of the problems posed January 27, the first was solved by "Auplume" and by John Snyder in Oconomowoc. John also solved the second problem, as did Liyang Liu; Liyang should stop by CMC 217 some time to pick up a B.B.O.P. item. Meanwhile, with any luck my solutions to the problems posed January 13 will be posted in the hallway outside CMC 217 by the time this *Gazette* appears. Good luck on the new problems!

- Mark Krusemeyer

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