



Goodsell Gazette

Carleton College
Northfield, MN 55057

The newsletter for the Carleton mathematics and statistics community

5 May 2017
Vol. 35, No. 18



Individual Comps Talks

The students who chose to present their independent comps during Spring Term are about to put the finishing touches on their presentations. They'll present in CMC 206 on Tuesday, May 9th and Thursday, May 11th. Stop by for a talk or two (or all of them!) and you'll be sure to learn something about the fields of mathematics and statistics you've never encountered!

Title: Persistent Homology and Viral Genetics

Speaker: Ari Bakke

Time: Tuesday, May 9th 3:30 - 4:00 p.m.

Abstract: Persistent homology is a relatively new branch of mathematics, only becoming greatly studied in the last decade or so, by people such as Gunnar Carlsson. It examines the topology - that is, the geometric structure - of a cloud of data points in the real-space of any dimension. Through it, we can make an educated guess as to what sort of shape these points have been taken from - more specifically, we can use homology to pinpoint clusters, holes, voids, and other such spatial features of a data set. Through this process, one particular research group was able to pinpoint instances of genetic recombination in viral DNA evolution, a lateral movement which the usual 'tree model' of evolution is unable to model. In effect, a homologous model gives us a much better model as to the evolution of viral DNA, amongst other uses.

Title: Linear Models - Variable Selection and Regularization Techniques

Speaker: Kristy Ketcham

Time: Tuesday, May 9th 4:00 - 4:30 p.m.

Abstract: When constructing a model, how do we determine the most accurate yet interpretable model? How do we fit models with many parameters and few observations while reducing variance and bias? Subset selection, shrinkage, and partial least squares are methods used to achieve these goals. During my presentation I will discuss the theory and algorithms behind these methods as well as the appropriate scenarios to use each type of method. Using my own data, I will also present examples of modeling using these methods and discuss how to conduct the analysis.

Title: The Game of Cops and Robbers on Finite Planar Graphs

Speaker: Ryan Salocks

Time: Tuesday, May 9th 4:30 - 5:00 p.m.

Abstract: The game of Cops and Robbers is a vertex pursuit game on graphs in which a set of cops plays against a single robber. The cops win by occupying the same vertex as the robber, and the robber wins by indefinitely avoiding the set of cops. Naturally, the strategic possibilities for both sides are largely determined by the properties of the graph they play on. We will consider how this game plays out on the class of finite planar graphs. In particular, we aim to establish an upper bound for the number of cops necessary to win the game in a finite number of rounds.

Title: Distribution of the Longest Run

Speaker: Ben White

Time: Tuesday, May 9th 5:00 - 5:30 p.m.

Abstract: Flip a coin 200 times. How likely is it to find at least one run of 7 consecutive heads? This talk discusses the concept and distribution of the longest run of an outcome in a sequence of trials, including the exact distribution of the longest run for both unbiased and biased coins, its expectation and variance, and the question of a limiting distribution. We will also discuss the application of this technique to matching DNA sequences, where it provides a structure to evaluate corresponding genetic sites in related species.

Title: Complex Chaos: Down the Rabbit Hole of the Mandelbrot Set

Speaker: Cisco Hayward

Time: Thursday, May 11th 3:30 - 4:00 p.m.

Abstract: Since its discovery in the 1980s, the Mandelbrot set shares a popularity comparable with Pi and e, and its dizzying architecture has been rendered in many a YouTube video. But what exactly is this mathematical monstrosity? Using the tools of Chaotic Dynamics, we will learn not just what the Mandelbrot set is, but *why* it is. The talk will start with a brief overview of orbits and iteration, introduce Fatou and Julia sets, construct the Mandelbrot set, and demonstrate some of its properties. We then modify our original construction in various ways to produce even stranger mathematical creatures, and we end with some discussion and unanswered questions for future students to address.

Title: An Introduction to the Invariant Subspace Problem

Speaker: Shilin Ma

Time: Thursday, May 11th 4:00 - 4:30 p.m.

Abstract: Though the Invariant Subspace Problem is not one of Millennium Prize Problems, it is a famous and important open problem in functional analysis. Some people even consider it to be the most fundamental question of operator theory. In general, the Invariant Subspace Problem questions whether there exists a non-trivial invariant subspace given a bounded linear operator. Knowledge of the invariant subspaces could help us understand the structure of the space. In the talk, I would like to show examples of interesting positive results in Banach space that have been achieved by studying certain classes of operators.

Title: The Isoperimetric Problem

Speaker: Margaret Bonnell

Time: Thursday, May 11th 4:30 - 5:00 p.m.

Abstract: The Isoperimetric Problem asks which figure, given all possible figures of a certain perimeter, encloses the greatest area. The problem is millennia old and has been solved in countless different ways since the Ancient Greeks first proved the answer to be a circle. We will examine some of the earlier proofs of this problem, which are based on Euclidean geometry, before questions regarding existence, as well as a desire to construct a proof using algebra in place of geometry began to define the problem in the 1800s. Finally we will examine Hurwitz's approach to the problem in the early 1900s, in which he applies the connection between Fourier series and geometry.

Title: The Coefficients of Cyclotomic Polynomials

Speaker: Pablo Martin Lucio Paredes

Time: Thursday, May 11th 5:00 - 5:30 p.m.

Abstract: Cyclotomic polynomials are some of the most studied and well understood polynomials in existence. They are present in number theory, abstract algebra, geometry, dynamical systems, and even helped advance research into Fermat's last theorem. This talk will focus on their coefficients, in particular, why a very logical conjecture to make for the coefficients of the n th polynomial has its first failure at $n=105$. Other fundamental properties of the polynomials will also be addressed, along with discussion of further results.

subject and how those results are used to solve applied problems, pulling examples from a wide range of disciplines.

Math 280: Statistical Consulting

Instructor: Katie St. Clair

Time: Thursdays only, 2/3c

Prerequisite: Math 245

Students will work on data analysis projects solicited from the local community. We will also cover the fundamentals of being a statistical consultant, including matters of professionalism, ethics and communication.

Math 295: Differential Forms and Vector Calculus

Instructor: Sam Patterson

Time: 3a

Prerequisite: Math 236

This course picks up where Calc III left off with vector calculus. In particular we will be concerned with such notions as divergence, curl, line and surface integrals and with extensions of Green's theorem to Gauss' Divergence theorem, and ultimately to Stokes' theorem in all its n -dimensional generality. This general Stokes' theorem, we will see, includes the Fundamental Theorem of Calculus, Green's theorem, and Gauss' theorem as particular cases. Our main tool for this further study of vector calculus will be differential forms—a modern mixture of geometric insight and algebra that will make precise what terms like dx and $dx dy$ really mean when written after integral signs. The prerequisites are simple just Calc III, Structures, and desire to learn more about vector calculus. The course is especially suitable for those students who have just completed Math 236.

Math 315: Topics Probability/Statistics: Time Series Analysis

Instructor: Andy Poppick

Time: 4a

Prerequisite: Math 245 and 275

Data that are observed over time often have a special kind of dependence structure: recent observations are more informative about the present than are past observations. Statistical models that assume independence (such as those you learned in Math 245) do not suffice in this setting. However, the unique structure of temporally varying data has motivated the development of a large class of techniques that broadly fall under the umbrella of time series analysis. This course will be an introduction to time series analysis, with a focus on modeling univariate quantitative data observed at evenly spaced intervals. The course will introduce you to two paradigms for thinking about time series—the so-called "time-domain" and "frequency-domain" approaches—and their connections to each other. We will explore some of the implications from these two perspectives, with an emphasis on model-building and data exploration in realistic applications.

Math 321: Real Analysis I

Instructor: Liz Sattler

Time: 5a

Prerequisite: Mathematics 236 or permission of the instructor

In calculus, we learned about some fundamental concepts in mathematics, like limits, continuous functions, and derivatives. Now that you've had some experience with the *structures* of mathematics, it's time for us to take a closer look at these concepts. What is a limit? What does it mean for a function to be continuous or differentiable? How can we rigorously define these terms and put them to good use? You may have noticed some epsilons and deltas in your calculus book, and in this course, we will learn how to use those epsilons and deltas to take a close up look at functions of a real variable. We will step out of our calculus comfort zone and examine oddly behaving functions, sequences, and sets (like the Cantor set) to understand the importance of formal definitions in calculus.

Math 332: Advanced Linear Algebra

Instructor: Eric Egge

Time: 2a

Prerequisite: Math 232 (or equivalent) and Math 236, or instructor permission

Much of Calculus is based on the idea that linear functions are simple, and that we can learn a lot about more

complicated functions by approximating them with linear functions. And it's true that linear transformations from the real line to itself are simple: they're just multiplication by a constant. But linear transformations from the plane to itself are more complicated: they include rotations, reflections, projections, and shears, as well as functions that are even harder to visualize, stretching the plane in one direction even as they compress it in another, or pushing all vectors into a certain line, which is itself mapped to zero. And that's just the plane! The story is even more complicated when we look at linear transformations from higher dimensional spaces to themselves.

One of our main goals in this course will be to learn as much as we can about linear transformations from a vector space to itself. You probably started this work in your first linear algebra course, when you learned about eigenvalues, eigenvectors, and how to use them to write some linear transformations in terms of diagonal matrices. You probably also saw that this is only partially effective: there are many linear transformations which are not diagonalizable. We will review these ideas, and build on them to describe how to understand even nondiagonalizable linear transformations. Along the way we will discover a marvelous connection between linear transformations and polynomials. We will also see how understanding linear transformations is inextricably tied to understanding vector spaces and their subspaces, and we may spend some time talking about interesting vector space constructions, such as dual spaces or inner product spaces.

We will take a somewhat more abstract approach to linear algebra than your first linear algebra course probably did. But don't worry if you think you've forgotten everything you learned in that course. In addition to reviewing eigenvalues, eigenvectors, and diagonalization, we will also review the other linear algebra ideas we need, including vector spaces and their subspaces, linear transformations, and how linear transformations, bases, and vector spaces are tied together.

Problems of the Fortnight

This is a special edition, so there are no problems this week! However, if you're interested, take a peek at last week's problems or even some from further back-- there are several back issues out near the whiteboard on the second floor of the CMC. Problems of the Fortnight will resume with our next regular issue-- next week!



Editors: **Elaina Thomas, Steve Kennedy**

Problems of the Fortnight: **Mark Krusemeyer**

Web & Subscriptions: **Sue Jandro**

