NUMS 2013

On Tuesday, October 29, several Carleton students will present their summer mathematics research at the 2013 Northfield Undergraduate Mathematics Symposium (NUMS). St. Olaf will be hosting this year’s event, and all of the talks will take place in Regents Hall of Natural Sciences, room 310. Come find out about the mathematics your classmates did this past summer, and enjoy the lively conversation at the pizza dinner. Even if you can’t stay for the whole event, you can still join for a couple of talks. The schedule of events is below.

There is a free bus to and from St. Olaf in the late afternoon and early evenings. Visit apps.carleton.edu/transportation/?option=localbus for more information. There is a map of the St. Olaf campus at www.stolaf.edu/map; Regents Hall of Natural Sciences is at the bottom, just to the right of center.

NUMS Schedule of Talks

3:30pm Finite Dynamical Systems: A Probabilistic Approach, Joey Dickens (St. Olaf)
Since the behavior of large finite dynamical systems is difficult to observe and characterize, we approach these number theoretic objects from a probabilistic point of view. By doing so, we develop expectations about large random finite dynamical systems. We find and prove several explicit and asymptotic formulas describing the growth of the set of periodic elements as set size becomes arbitrarily large. We conclude with several additional conjectures concerning the asymptotic behavior of the set of periodic elements when we have a d-to-one function.

3:55pm Evaluating an Adaptive Clinical Trial with Quantitative Endpoints, Sample Size Re-estimation, Sequential Monitoring for Efficacy, and Monitoring for Futility, Harrison Reeder (Carleton)
Clinical trials are an essential component to modern evidence-based medicine, and biostatisticians are continually developing new trial designs to maximize their safety, efficiency, and value. This research focuses on exploring the characteristics of a particular Phase II trial design with three key properties: adaptive sample size recalculation, interim monitoring for efficacy, and monitoring for futility. Using a simulation study, we evaluate the performance of the design and compare the value of trials with some or all of these characteristics. Our comparison measures include the accuracy of the designs' estimation of treatment effect, the error rates of the trials, as well as the robustness of the designs to inaccurate assumptions about the treatment. Our research also compares the merit of three different interim monitoring schemes in this design, observing that O'Brien-Fleming boundaries are most suitable. Our overall findings conclude that compared to a simple design with interim monitoring for efficacy, the complete design is an improvement; our design shows comparable performance under most conditions and improved overall performance under conditions where initial design estimates are inaccurate.

4:20pm An Analysis of the Trojan Y-Chromosome Method of Invasive Species Management, Jared Brown (St. Olaf)
Management of invasive species towards the goal of preserving native biodiversity and preventing economic damage has traditionally been one of the most challenging problems faced by modern ecological scientists. The introduction of modified members of the invasive
species, carrying extra trojan Y-chromosomes, may have a much less harmful, and thus less expensive technique for controlling or eliminating wild populations of undesired, sexually reproducing organisms. This paper (talk) presents both deterministic and Stochastic models of the reaction of the wild population to such trojan introduction. Results upon arbitrary species parameters support the potential validity of this technique, and give insight into more environmentally specific interactions.

4:45pm Volumes of Hyperbolic Knot Complements, Martin Bobb (Carleton)
A knot is an embedding of a circle in a sphere. Many knots, including 2-bridge knots, have complements with a hyperbolic structure determined by the knot. We explore the hyperbolic volumes of knot complements for 2-bridge links obtained by Dehn fillings. We build on work by Purcell to more accurately explore how the geometry of universal covers and gluing operations affect hyperbolic volumes.

5:10pm Digraphs, Zero Forcing, and Maximum Nullity, Cora Brown (Carleton) and Nathanael Cox (St. Olaf)
A simple digraph, $\Gamma$, is a set of vertices and arcs whose elements are ordered pairs of vertices. The zero forcing number for a digraph $\Gamma$ is the minimum number of blue vertices needed to force all the vertices of $\Gamma$ to become blue by the color change rule. This rule states that for $\Gamma$ with all vertices colored blue or white, a blue vertex, $v$, can force an adjacent white neighbor, $w$, to become blue if $w$ is the only white out-neighbor of $v$. The maximum nullity of a digraph, $M(\Gamma)$, is the maximum nullity of any of these matrices described by $\Gamma$. We will present results on maximum nullity, zero forcing number, and other properties of digraphs including techniques for finding the minimum rank of digraphs, results for oriented graphs (digraphs that allow no doubly directed arcs), and results for directed graphs in general.

5:30pm Pizza Dinner and Conversation in Regents Hall of Natural Sciences, room 356

6:00pm Difference Set Transfers, Dylan Peifer (Carleton)
Given a finite group $G$ of order $v$, a subset $D$ of $G$ is called a $v, k, \lambda$-difference set if $|D| = k$ and the set \{ $d_i d_j^{-1} \mid d_i, d_j \in D$ \} contains $\lambda$ copies of each nonidentity element of $G$. The fundamental question in the study of difference sets is determining which groups contain difference sets and which do not, and a related question involves finding all difference sets in a group or set of groups. Though exhaustive search can easily determine all difference sets in groups of order 16, there are still many patterns to be found in what we term difference set transfers – where a difference set in one group of order 16 can be transferred to a difference set in a different group of order 16 using power-commutator presentations of these groups. In this talk we will examine and prove many of the difference set transfers found in groups of order 16 and apply transfers to groups of order 64 and 144, where exhaustive search is infeasible and other standard methods for finding difference sets fail.

6:25pm Nosé-Hoover Thermostats, Lora Weiss, (St. Olaf)
The equilibrium statistical properties of molecular systems are important to applied subjects such as biology, chemistry, computational physics and materials science. These equilibrium statistical properties are obtained as phase space integrals that depend on $q$ as the position of the system, $p$ as the momentum of the system, and have $H(q, p)$ as the total energy of the system. In 1984 S. Nosé introduced a thermostat to mimic the effect of a heat bath on a mechanical system. W. Hoover simplified this model and showed that even for the simple harmonic oscillator the system can exhibit complicated dynamics. In this presentation, we attempt to find exact solutions of the Nosé-Hoover thermostat; we look for periodic solu-
tions to make a conjecture about the existence of invariant tori; we determine orbit averages along the solution curves; we analyze various numerical methods to solve the system; and we look at the existence of a first integral for the system, building on the work of Legoll, Luskin, and Moeckel.

6:50pm Cayley Graphs and the Cayley-Isomorphism Property, Greg Michel (Carleton)

For a finite group \( G \) and a subset \( S \) of \( G \), the Cayley graph \( \text{Cay}(G,S) \) is the graph whose vertex set is \( G \) such that two vertices \( x \) and \( y \) are adjacent if \( x^{-1}y \) is in \( S \). A Cayley graph \( \text{Cay}(G,S) \) is called a CI-graph if for any \( \text{Cay}(G,T) \) that is graphically isomorphic to \( \text{Cay}(G,S) \), there is a corresponding group automorphism \( \sigma \) of \( G \) with \( \sigma(S) = T \). A finite group \( G \) is called a CI-group if every Cayley graph of \( G \) is a CI-graph. We show that \( G \) is not CI if it admits a non-CI subgroup or if it admits two non-isomorphic subgroups of the same order. We show that this completely classifies the effect of subgroups on non-CI Abelian groups. That is, if an Abelian group is non-CI and its subgroups do not meet the conditions above, then any non-CI graph will be connected.

7:15pm Non-Mass Action Modeling for the Binding of Phosphorylated Gli1 with SUFU, Taisa Kushner (St. Olaf)

Our goal in this research project was to construct a mathematical model to accurately simulate Gli1-Erk2- SUFU interactions that were observed biologically. Gli1 is a protein associated with the Hedgehog (Hh) signaling pathway, which is involved in embryonic development and stem cell differentiation. Over-expression of the Gli1 protein has been linked to many cancers, most notably glioblastoma multiforme (GBM), the most common and aggressive brain tumor. To create our model, we proposed the novel incorporation of Holling Type-II interactions from ecology into a biochemical model constructed using differential equations and a multiple-time-scale system. We compare this with other models, such as the mass-action protein interaction model and a Gli1-dimerization model and show that these are insufficient for explaining the observed dynamics.

Winter Term
Course Descriptions

Math 206 A Tour of Mathematics (1 credit; S/Cr/NC)
Instructors: Many of us
Time: Friday only, 6a
Description: Are you considering a math major, but wondering just what will follow after all the calculus and linear algebra, or where the frontiers of mathematical knowledge are to be found? Are you already a major who would enjoy some fresh perspectives on, and new insights into, your chosen subject? Come join us for a series of lectures on a variety of mathematical topics, with emphasis on exciting ideas, concepts and results rather than on systematic coverage of any particular subject (we have other courses for that). Although this course has been offered yearly, there should be no overlap in lectures with last winter’s offering, so you can “repeat” if you took the 2013 Tour.

Math 236 Mathematical Structures
Instructor: Eric Egge
Time: 3a
Prerequisite: Math 232 / consent of instructor
Description: If mathematics is a city, then this course is all about how we build its various parts. We'll study set theory, formal logic, and axiomatic systems, which are the raw materials that go into every building. We'll learn about techniques for discovering (or inventing) proofs, common methods of proof, and how to write good proofs; these are the construction methods we use to build everything in the city. And we'll study some fascinating problems and results that everyone should
know, such as the many sizes of infinity; these are our city's major landmarks, our Opera House, Temple Mount, Golden Gate Bridge, or Forbidden City.

Math 236 is the first course that suggests what being a math major (as opposed to a math user) is all about. If you are considering majoring in math, then this course should help you decide. This course is also a prerequisite for many upper level mathematics courses, so taking it gives you the keys to a whole new mathematical world.

Math 241 Ordinary Differential Equations
Instructor: Mark Krusemeyer
Time: 5a
Prerequisite: Math 232 / consent of instructor
Description: In calculus you may well study separable first-order differential equations for a bit, but that's just the tip of the iceberg! In any field where mathematics is applied, you are likely to find equations relating unknown functions and their derivatives. Over the centuries, following the lead of Newton, Leibniz, and the Bernoullis, mathematicians have come to grips with many such equations. Naturally, they prefer to get exact solutions if possible, and we'll look at some of the systematic methods (and a few of the clever ad hoc tricks) that have been developed to find solutions. On the other hand, there are times when finding an exact solution is too difficult, or even potentially misleading – for instance, because the mathematical model that leads to the differential equation is imprecise to begin with. In such cases, it is often best to concentrate on the qualitative behavior of solutions; for example, you might try to predict what will happen in the long run.

In this course, you'll find plenty of calculus-style computation, including ample opportunity to brush up on your techniques of integration (Mathematica can help with some of that), but also a few theoretical discussions, some geometric ideas, and a bit of mathematical modeling. The textbook we'll be using, which was written by a close (younger!) relative, does not presuppose much linear algebra, but concepts from linear algebra, ranging from vector spaces of functions through linear transformations and kernels to eigenvalues and eigenvectors, will be mentioned and used with some regularity in class.

Math 245 Applied Regression Analysis
Instructor: Laura Chihara
Time: 3a
Prerequisite: Math 215 (AP statistics 4/5) or Math 275
Description: On the night of January 27, 1986, engineers at Morton Thiokol teleconferenced with engineers and managers at the Marshall Space Flight Center and Kennedy Space Center to determine whether it was too cold (31°F) to launch space shuttle Challenger. Data from previous flights seemed to suggest that temperature had an effect on the integrity of the O-ring seals on the booster rockets, but the final recommendation was to launch the Challenger on schedule. Could a statistical analysis of the pre-accident data have predicted the catastrophic failure of the shuttle? In this class, we will investigate the Challenger data and in general, learn statistical model building and model checking techniques. We will use the software package R to aid in the modeling.

Math 275 Introduction to Statistical Inference
Instructor: Laura Chihara
Time: 4a
Prerequisite: Math 265
Description: Statistics is the art and craft of studying data and understanding variability. Though mathematics (in particular, probability) governs the underlying theory, statistics is driven by applications to real problems. We will cover basic statistical inference as well as modern computational approaches, all in the context of investigating interesting questions that arise in scientific and public policy settings. We will use the software package R.
Math 244 Geometries
Instructor: Stephen Kennedy
Time: 2a
Prerequisite: Math 236
Description: Goethe described it as “the fountain of all truth.” Plato said it’s how god thinks. Poincare said it’s how you think. Edna St. Vincent Millay called it, “Beauty bare.” It has been inspiring poets, philosophers, scientists and schoolboys and schoolgirls for 3000 years. It's geometry. Come see what all the fuss is about. We will start with a quick revisit to Euclid's Elements, quickly skim over a couple of millennia of progress and then wallow in the creations of the last few centuries. We will learn some fabulous theorems about circles and triangles that, had they been known, would have delighted Euclid. Then we will wander in non-Euclidean space and learn some stuff that would have absolutely flipped him out – it should have the same effect on you. Required for prospective high-school teachers, recommended for anyone interested in really cool ideas.

Math 331 Real Analysis II
Instructor: Gail Nelson
Time: 4a
Prerequisite: Math 321
Description: In this second course of the analysis sequence, we will delve even deeper into the properties of functions. Specific topics will include Lebesgue measure, the Lebesgue integral, an introduction to general measure theory, and Banach and Hilbert spaces. Not only is this your chance to “integrate” your knowledge of functions, it is also an opportunity to better your understanding of the legal interchange of limit operations. And, of course, the Cantor set will make its usual appearance! The flavor of the course will be similar to a graduate-level course in analysis. If there is a possibility that you are headed for graduate school in mathematics or a related field, this course comes highly recommended.

Math 342 Abstract Algebra I
Instructor: Jack Goldfeather
Time: 3a
Prerequisite: Math 236 / consent of instructor
Description: Abstract algebra is a fascinating area of pure mathematics that has applications in several directions, which you might not expect. For example, the ways of describing symmetry that we will look at are used in theoretical physics (among other things, to predict the existence of elementary particles) and in quantum chemistry. But they can also be used in the study of artistic patterns, such as wallpaper patterns or Escher’s “regular division drawings.” We will explore exotic settings for arithmetic, once thought to be of purely theoretical interest, which in the last sixty years have found numerous applications to such things as the design of error-correcting codes. If time permits, we may see how famous construction problems from ancient Greek geometry were shown to be unsolvable, about 2000 years after they were first proposed! All this will be done using algebraic structures such as groups, rings, and fields, but we’ll see what those technical terms mean during the course itself. To get a preview, have a look at the highly entertaining textbook we’ll use, Gallian’s Contemporary Abstract Algebra.

Math 395 Topics in Algebraic Number Theory
Instructor: Rafe Jones
Time: 5a
Prerequisite: Math 342, an equivalent Budapest or Moscow Semester in Mathematics course, or consent of the instructor
Description: The theory of algebraic numbers has grown out of a famous (and famously useful) mistake. In an effort to prove the elusive Last Theorem of Fermat, the French mathematician Lamé proposed a method in 1847 that appeared to slay the monster. But no sooner had he given his talk claiming a proof than Liouville pointed out that it relied on generalizations of familiar properties of the integers to more exotic rings. These rings, known as rings of algebraic integers, turned out to have many...
tricks up their sleeves, most notably a failure in many cases of the unique factorization property that the usual integers enjoy. This failure led to the failure of Lamé's proof, but also led to the intense study of rings of algebraic integers, which continues to this day.

In this class we'll embark on our own study of algebraic integers. We'll see that, while unique factorization may fail, it can be salvaged if we consider ideals rather than elements: every ideal has a unique factorization into a product of prime ideals. We'll introduce the class group, which is a group whose size measures the extent to which unique factorization of elements fails, and if time permits we'll use a marvelous geometric argument to prove that it is a finite group. Depending on time and the interests of the class, we may also examine cyclotomic fields (and in particular Kummer's partial salvaging of Lamé's argument), some of the ideas involved in Wiles' eventual complete proof of Fermat's Last Theorem, units in number fields, and the Chebotarev density theorem. Presentations by students will be an important component of the course, and account for a significant portion of the course grade.

**PROBLEM OF THE WEEK**

Suppose $f$ is a polynomial in $n \geq 2$ variables of degree $d \leq n - 1$. (For example, $x^2 + wyz$ is of degree 3 but has 4 variables.) Prove that

$$
\sum_{\delta_1, \ldots, \delta_n \in \{0, 1\}} (-1)^{\delta_1 + \cdots + \delta_n} f(\delta_1, \ldots, \delta_n) = 0
$$

where the sum is over the $2^n$ possible ways to assign the values 0 and 1 to each of the variables $\delta_1, \ldots, \delta_n$.

**Acknowledgments**

Jacob Spear submitted a solution to the two-week old tetrahedron problem, and Isaac Garfinkle and Leo Betthauser correctly solved last week's problem. Jacob and Leo (by lottery) should stop by Andrew's office for a prize from the BBOP! Additionally, John Snyder in Oconomowoc submitted an interesting *Mathematica* analysis of this week's problem using the theory of Gröbner bases, and pseudonymous submitter "Cyclist" provided an elegant formal solution.